Infinite Stream Estimation under Personalized w-Event Privacy

Leilei Du
Hunan University
Changsha, China
leileidu@hnu.edu.cn

Peng Cheng
Tongji University
Shanghai, China
cspcheng@tongji.edu.cn

Lei Chen
HKUST (GZ) & HKUST
Guangzhou & HK SAR, China
leichen@cse.ust.hk

Heng Tao Shen
Tongji University & UESTC
Shanghai & Chengdu, China
shenhengtao@hotmail.com

Xuemin Lin
Shanghai Jiaotong University
Shanghai, China
xuemin.lin@gmail.com

Wei Xi
Xi'an Jiaotong University
Xi'an, China
xiwei@xjtu.edu.cn

ABSTRACT

Streaming data collection is indispensable for stream data analysis, such as event monitoring. However, publishing these data directly leads to privacy leaks. w-event privacy is a valuable tool to protect individual privacy within a given time window while maintaining high accuracy in data collection. Most existing w-event privacy studies on infinite data stream only focus on homogeneous privacy requirements for all users. In this paper, we propose personalized w-event privacy protection that allows different users to have different privacy requirements in private data stream estimation. Specifically, we design a mechanism that allows users to maintain constant privacy requirements at each time slot, namely Personalized Window Size Mechanism (PWSM). Then, we propose two solutions to accurately estimate stream data statistics while achieving $\boldsymbol{w}$-Event $\boldsymbol{\epsilon}$-Personalized Differential Privacy (($\boldsymbol{w}$,$\boldsymbol{\epsilon}$)-EPDP), namely Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA). PBD always provides at least the same privacy budget for the next time step as the amount consumed in the previous release. PBA fully absorbs the privacy budget from the previous k time slots, while also borrowing from the privacy budget of the next k time slots, to increase the privacy budget for the current time slot. We prove that both PBD and PBA outperform the state-of-the-art private stream estimation methods while satisfying the privacy requirements of all users. We demonstrate the efficiency and effectiveness of our PBD and PBA on both real and synthetic datasets, compared with the recent uniformity w-event approaches, Budget Distribution (BD) and Budget Absorption (BA). Our PBD achieves 68% less error than BD on average on real datasets. Besides, our PBA achieves 24.9% less error than BA on average on synthetic datasets.

PVLDB Reference Format:
Leilei Du, Peng Cheng, Lei Chen, Heng Tao Shen, Xuemin Lin, and Wei Xi. Infinite Stream Estimation under Personalized w-Event Privacy. PVLDB, 18(6): 1905 - 1918, 2025.
doi:10.14778/3725688.3725715

PVLDB Artifact Availability:

Proceedings of the VLDB Endowment, Vol. 18, No. 6 ISSN 2150-8097.
doi:10.14778/3725688.3725715

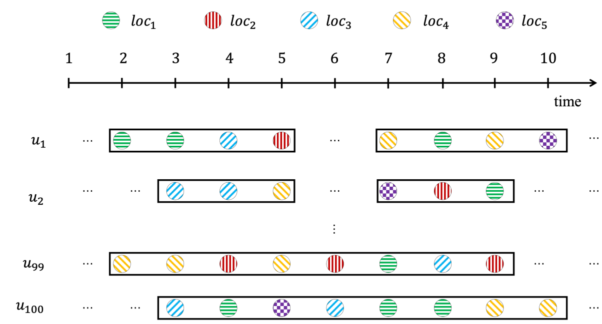


Figure 1: Different event window sizes for different time slots.

The source code, data, and/or other artifacts have been made available at https://github.com/dulei715/DynamicWEventCode.

1 INTRODUCTION

With the popularity of smart devices and high-quality wireless networks, people can easily access the internet and utilize online services. They continuously report data to platforms and receive services like log stream analysis [34], event monitoring [19], and video querying [27]. To provide better services, these platforms collect data and conduct real-time analysis over aggregated data streams.

However, collecting stream data directly poses severe privacy risks, causing users to refuse communication with platforms. For instance, an AIDS patient may decline to participate in an investigation due to privacy concerns [18]. To resolve this conflict, differential privacy (DP) is proposed to protect individual privacy while ensuring accurate data estimation [11].

Recently, w-event privacy based on DP has emerged for private stream data collection and analysis [29, 30, 33]. It effectively protects the privacy of w consecutive related events while offering accurate stream statistics. However, different users may have different privacy requirements. For instance, entertainers may be reluctant to reveal too much about their locations (i.e., large w-event size), while street artists may be willing to expose their locations (i.e., small w-event size) for more attention. Thus, if we fix the window size w for all users, it is hard to make everyone satisfied.

We illustrate an example of online car-hailing shown in Figure 1.

Example 1. *Consider a scenario with* 100 *drivers* $U = \{u_1, ..., u_{100}\}$ *who share their locations from* $\{loc_1, ..., loc_5\}$ *at each time slot. Each driver* u_i *is protected by* w_i*-event privacy, meaning* *their location data is safeguarded through ϵ-DP across at least w_i consecutive time slots, where ϵ represents their required privacy protect stength. For example, u_1 requires location protection across any* 4 *consecutive time slots, while u_{99} and u_{100} need protection across any* 8 *consecutive time slots. For the drivers $u_i \in U \backslash \{u_{99}, u_{100}\}$, the window size does not exceed* 4.

Under traditional w-event privacy, satisfying all drivers' privacy needs requires setting the event window size to the maximum value (i.e., $w = 8$) and making full use of the privacy budget to achieve high utility while maintaining 8*-event privacy. Let AE_{avg} denote the average square error at each time slot, defined as the variance when adding Laplace noise (i.e., $AE_{avg} = 2b^2 = 2 \times \left(\frac{1}{\epsilon/w}\right)^2$). With a total privacy budget ϵ of* 1 *and using the Uniform method [22], the average square error at each time slot under* 8*-event privacy is $AE_{avg} = 2 \times (\frac{w}{\epsilon})^2 = 128$. However, the first* 98 *drivers do not actually need* 8*-event privacy. By setting the window size to $w = 4$ and using the threshold method [20] (or the sample method [20]), we can achieve $AE_{avg} \approx 2 \times (\frac{w}{\epsilon})^2 = 32$, which is significantly lower than the error from traditional* 8*-event privacy.*

In this paper, we define the Personalized w-Event Private Publishing for Infinite Data Streams (PWEPP-IDS) problem to model personalized privacy requirements in stream data publication. To solve PWEPP-IDS, there are two challenges: 1) effectively unifying the privacy budget across all users into a single value to maximize publication utility; 2) effectively distribute each user's personalized privacy budget to their personalized window size to maximize publication utility.

To improve publication utility, we address PWEPP-IDS using the centralized DP model [11], which requires a single centralized privacy budget for publishing statistics at each time slot. However, users have different personalized privacy budgets (not the same budgets), traditionally, satisfying everyone's privacy requirements means selecting the minimum budget among all users, which results in the lowest utility. *How to use a privacy budget higher than the minimum one to achieve higher utility while satisfying the privacy requirement of user with the minimum privacy?* It seems unachievable at a glance. We solve this challenge through elaborately applying the Sampling Mechanism [20]. Our method theoretically guarantees that even though the selected unified privacy budget is higher than the minimum privacy budget, no privacy leakages for any users exist.

Intuitively, time slots with higher rates of change contain more valuable information and are thus more important. To maximize utility, we need to allocate large privacy budgets to publications at these important time slots while approximating others (allocating none). *How to identify important time slots and allocate privacy budgets to achieve maximum publication utility?* To address this challenge, we design two methods: Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA). Both methods identify important time slots by measuring the dissimilarity between current and historical statistics. The key difference lies in their assumption: PBD assumes stream data has a stable and high rate of change. Thus, it allocates budgets to each publication in an exponentially decreasing fashion per window. PBA, however, assumes stream data has an unstable and low rate of change. Thus, it skipps or approximates many less important publications. Besides, it maximizes the accuracy of important publications by utilizing unused budget from skipped publications while nullifying future time slot budgets. We prove that both PBD and PBA satisfy $\boldsymbol{w}$-Event $\boldsymbol{\epsilon}$-Personalized Differential Privacy (($\boldsymbol{w}$,$\boldsymbol{\epsilon}$)-EPDP) and provide their average error upper bounds. We summarize our contributions as follows.

- We formally define $\boldsymbol{w}$-Event $\boldsymbol{\epsilon}$-Personalized Differential Privacy for PWEPP-IDS in Section 3.
- We propose Personalized Window Size Mechanism (PWSM) and two methods, namely Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA), to support personalized $\boldsymbol{w}$-event privacy with theoretical analyses in Section 4.
- We test our methods on both real and synthetic data sets to demonstrate their efficiency and effectiveness in Section 5.

2 RELATED WORK

2.1 Data Stream Estimation under Differential Privacy

Based on the privacy model, there are two types of data stream estimation methods: centralized differential privacy [11] (CDP) based methods and local differential privacy [4] (LDP) based methods.

Data Stream Estimation under CDP. Dwork et al. first address the problem of Differential Privacy (DP) on data streams [13]. They define two types of DP levels: *event-level differential privacy* (event-DP) and *user-level differential privacy* (user-DP).

In event-DP, each single event is hidden in statistic queries. Dwork et al. focus on the finite event scenarios and propose a binary tree method to achieve high statistical utility while maintaining event-DP [13]. Chan et al. extend it to infinite cases, and produce partial summations for binary counting [7]. Dwork et al. introduce a cascade buffer counter that updates adaptively based on stream density [12]. Bolot et al. propose *decayed privacy* which reduces the privacy costs for past data [6]. Chen et al. develop PeGaSus, a perturb-group-smooth framework for multiple queries under event-DP [8]. However, event-DP assumes all element in a stream are independent, making it unsuitable for correlated data stream publishing.

In user-DP, all events for each user are hidden in statistic queries. Fan et al. propose the FAST algorithm with a sampling-and-filtering framework, counting finite stream data under user-DP [17]. Cummings et al. address heterogeneous user data, estimating population-level means while achieving user-DP [9]. However, they only consider finite data. Offering user-DP for infinite data requires infinite perturbation, leading to poor long-term utility [22].

To bridge the gap between event-DP and user-DP, Kellaris et al. propose w-event DP for infinite streams [22]. It ensures ϵ-DP for any group of events within a time window of size w. They introduce two methods, *Budget Distribution* and *Budget Absorption*, to optimize privacy budget use and estimate statistics effectively. However, neither method handles stream data with significant changes. Wang et al. apply the w-event concept to the FAST method, proposing a multi-dimensional stream release mechanism called *ResueDP*, which achieves accurate estimation for both rapid and slow data stream changes [30]. A limitation of all these methods is their reliance on a trusted server to ensure privacy.

Data Stream Estimation under LDP. To overcome the dependence on a trusted server, LDP [4] has recently been proposed and adopted by many major companies such as Microsoft, Apple and Google. Erlingsson et al. introduce RAPPOR to estimate finite streams under LDP [16]. They design a two-layer randomized response mechanism (i.e., permanent randomized response and instantaneous randomized response) to protect each individual's data. However, RAPPOR is limited to uncorrelated stream data. To address the problem of correlated time series data, Erlingsson et al. develop a new privacy model that introduces *shuffling* to amplify the LDP privacy level [15]. However, this model only suits finite stream data. Joseph et al. propose THRESH for evolving data under LDP [21], which consumes privacy budget at global update time slots selected by users' LDP voting. However, it is not applicable to infinite streams as it assumes a fixed number of global updates. Wang et al. extend event-level privacy from CDP to LDP and design the efficient ToPL method under event LDP [31]. Nevertheless, event-level LDP focuses solely on event-level privacy, lacking privacy protection for correlated data in streams. Bao et al. propose an (ϵ, δ)-LDP method (called CGM) for finite streaming data collection using the analytic Guassian mechanism, but requires periodic privacy budget renewal [3]. Ren et al. introduce LDP-IDS for infinite streaming data collection and analysis under w-event LDP [29]. They propose two budget allocation methods and two population allocation methods, bridging the gap between event LDP and user LDP while improving estimation accuracy. However, all these methods cannot be adopted to support personalized event window sizes.

2.2 Non-Uniformity Differential Privacy

Recently, some studies address the non-uniform privacy requirements among items (table columns) or records (table rows) [28].

Alaggan et al. first examine scenarios where each database instance comprises a single user's profile [1]. They focus on varying privacy requirements for different items and formally define Heterogeneous Differential Privacy (HDP).

Jorgensen et al. investigate the privacy preservation for individual rows, introducing Personalized Differential Privacy (PDP) [20]. They design two mechanisms leveraging non-uniform privacy requirements to achieve better utility than standard uniform DP. Kotsogiannis et al. recognize that different data have different sensitivity, then define One-side Differential Privacy (OSPD) and propose algorithms that truthfully release non-sensitive record samples to enhance accuracy in DP-solutions [23].

Andrés et al. introduce a novel non-uniform privacy concept called Geo-Indistinguishability (Geo-I), where the privacy level for any point increases as the distance to this point decreases [2]. Wang et al. [32] and Du et al. [10] explore PDP in spatial crowdsourcing, and develop highly effective private task assignment methods to satisfy diverse workers' privacy and utility requirements. Liu et al. investigate HDP in federated learning [26]. They assume different clients hold DP budget and divide them into private and public parts, then propose two methods to project the "public" clients' models into "private" clients' models to improve the joint model's utility. However, all above studies are not suitable for stream data.

Table 1: Summary for related work.

Model Types		Methods	Is infinite and correlated	Is personalized privacy
Centralized DP	event-level privacy	Finite B-tree [13]	✗	✗
		Infinite B-tree [7]	✗	✗
		Adaptive-density Counter [12]	✗	✗
		Decayed Privacy [6]	✗	✗
		PeGaSus [8]	✗	✗
	user-level privacy	FAST [17]	✔	✗
		Private heterogeneous mean estimation [9]	✔	✗
	w-event privacy	BD & BA [22]	✔	✗
		ResuseDP [30]	✔	✗
Local DP	event-level privacy	RAPPOR [16]	✗	✗
		ToPL [31]	✗	✗
	user-level privacy	Shuffling LDP [15]	✔	✗
		THRESH [21]	✔	✗
		CGM [3]	✔	✗
	w-event privacy	LDP-IDS [29]	✔	✗
Item heterogeneous		HDP [1]	✗	✗
Record heterogenous		PDP [20]	✗	✔
		OSDP [23]	✗	✔
		Geo-I [2]	✗	✔
		PWSM, VPDM [32]	✗	✔
		PUCE, PGT [10]	✗	✔
		PFA, PFA+ [26]	✗	✔
Our mechanisms			✔	✔

In this paper, we propose Personalized Window Size Mechanism (PWSM) with two implementation methods: Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA). Our approach extends traditional w-event privacy mechanisms by introducing $\boldsymbol{\epsilon}$-personalized differential privacy methods to support personalized privacy requirements. This enhancement enables our mechanism and methods to handle both infinite correlated data streams and personalized privacy requirements, building upon the foundations of traditional w-event privacy mechanisms.

3 PROBLEM SETTINGS

In this section, we first introduce key concepts, including data streams. Next, we present the new definition of $\boldsymbol{w}$-Event $\boldsymbol{\epsilon}$-Personalized Differential Privacy. Finally, we provide the problem definition: Personalized w-Event Private Publishing for Infinite Data Streams (PWEPP-IDS). Table 2 outlines the notations used throughout this paper.

3.1 Data Stream

Definition 1. (Data Stream [22]). Let $D_t \in \mathcal{D}$ be a database with d columns and n rows (each row representing a user) at t-th time slot. The infinite database sequence $S = [D_1, D_2, \ldots]$ is called a data stream, where $S[t]$ is the t-th element in S (i.e., $S[t] = D_t$).

For any data stream S, its substream between time slot t_l and t_r (where $t_l < t_r$) is noted as $S_{t_l,t_r} = [D_{t_l}, D_{t_l+1}, \ldots, D_{t_r}]$. For $t_l = 1$, we denote $S_t = [D_1, D_2, \ldots, D_t]$ and call it the *stream prefix* of S.

Definition 2. (Data Stream Count Publishing [22]). Let $Q : \mathcal{D} \rightarrow \mathbb{R}^d$ be a count query. Then, $Q(S[t]) = Q(D_t) = \boldsymbol{c}_t$ is the count data to be published at time slot t, where $\boldsymbol{c}_t(j)$ represents the count of the j-th column of D_t. The infinite count data series $[\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots]$ is called a data stream count publishing.

3.2 w-Event ϵ-Personalized Differential Privacy

Definition 3. (w-Neighboring Stream Prefixes [7, 22]). Let w be a positive integer, two stream prefixes S_t, S'_t are w-neighboring (i.e., $S_t \sim_w S'_t$), if

Table 2: Notations.

Notations	Description
$\mathcal{D}$	the database domain
D_t	a database at time slot t
S	a data stream
u_i	the i-th user
$\boldsymbol{x}_{i,t}$	u_i's data at time slot t
$\boldsymbol{c}_t$	a real statistical histogram at time slot t
$\boldsymbol{r}_i$	an estimation statistic histogram at time slot t
w_i	u_i's privacy window size
ϵ_i	u_i's privacy budget

(1) for each $S_t[k], S'_t[k]$ such that $k \le t$ and $S_t[k] \ne S'_t[k]$, it holds that $S_t[k]$ and $S'_t[k]$ are neighboring [22] in centralized DP, and

(2) for each $S_t[k_1]$, $S_t[k_2]$, $S'_t[k_1]$, $S'_t[k_2]$ with $k_1 < k_2$, $S_t[k_1] \ne S'_t[k_1]$ and $S_t[k_2] \ne S'_t[k_2]$, it holds that $k_2 - k_1 + 1 \le w$.

Definition 4. ($\boldsymbol{w}$-Event $\boldsymbol{\epsilon}$-Personalized Differential Privacy, ($\boldsymbol{w}$,$\boldsymbol{\epsilon}$)-EPDP). Let $\mathcal{M}$ be a mechanism that takes a stream prefix of arbitrary size as input. Let $\mathcal{O}$ be the set of all possible outputs of $\mathcal{M}$. Given a universe of users $U = \{u_1, u_2, \ldots, u_{|U|}\}$, then $\mathcal{M}$ is $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP if $\forall O \subseteq \mathcal{O}$, $\forall w_i \in \boldsymbol{w}$ and $\forall S_t, S'_t$ satisfying $S_t \sim_{w_i} S'_t$, it holds that

$$\Pr[M(S_t) \in O] \le e^{\epsilon_i} \Pr[M(S'_t) \in O],$$

where $u_i \in U$ requires w_i-event privacy and ϵ_i denotes u_i's privacy budget requirement within w_i continuous events.

We denote the pair (w_i, ϵ_i) as u_i's *privacy requirement*. Specifically, when $w_i = 1$, it collapses as $\boldsymbol{\epsilon}$-Personalized Differential Privacy ($\boldsymbol{\epsilon}$-PDP) [20]. Besides, when (w_i, ϵ_i) becomes constant (i.e., (w, ϵ)), it collapses as w-Event Privacy[22, 29].

3.3 Definition of PWEPP-IDS

Given a data stream S, the server aims to obtain the data stream count publishing, denoted as $\boldsymbol{c} = [\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots]$. However, to protect user privacy, the server only receives the obfuscated version of the data stream, S', and subsequently publishes the estimated data stream count (i.e., estimation count), denoted as $\boldsymbol{r} = [\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots]$. We now define the problem as follows.

Definition 5. (PWEPP-IDS). Given a user set $U = \{u_1, u_2, ..., u_n\}$, each u_i holds a privacy requirement pair (w_i, ϵ_i) and a series data $\boldsymbol{x}_{i,t}$ for $t \in \mathbb{N}^+$. All the $\boldsymbol{x}_{i,t}$ for $u_i \in U$ at time slot t form D_t. All the D_t form an infinite data stream $S = [D_1, D_2, \ldots]$. PWEPP-IDS is to publish an obfuscated histogram $\boldsymbol{r} = [\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots]$ of S at each time slot t achieving $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP with the error between $\boldsymbol{r}$ and $\boldsymbol{c}$ minimized, namely $\forall T \in \mathbb{N}^+$:

$$\min_{\epsilon_\theta} \sum_{t \in [T]} \|\boldsymbol{r}_t - \boldsymbol{c}_t\|_2^2$$

$$s.t. \sum_{k=\min(t-w_i+1,1)}^{t} \epsilon_{i,k} \le \epsilon_i, \quad \forall u_i \in U$$

where $\epsilon_{i,k}$ indicates the privacy budget at time slot k.

4 PERSONALIZED WINDOW SIZE MECHANISM

In this section, we analyze the errors in reporting obfuscated data stream counts and introduce Optimal Budget Selection (OBS) method to minimize these errors. We then propose Personalized Window Size Mechanism (PWSM) to address PWEPP-IDS. The core idea of PWSM is to select the optimal privacy budget $\epsilon_{opt}(t)$ and report obfuscated count results that satisfy $\epsilon_{opt}(t)$-DP at each time slot t.

4.1 Reporting Errors

For each time slot, we use the Sampling Mechanism (SM) [20] to satisfy all users' privacy requirements (i.e., achieving $\boldsymbol{\epsilon}$-PDP). The SM consists of two steps: *sample* (SM_s) and *disturb* (SM_d). In SM_s, the server first sets a privacy budget threshold ϵ_θ, then constructs a sampling subset D_S by appending items x_i with $\epsilon_i \ge \epsilon_\theta$ to D_S, while sampling other items x_j with $\epsilon_j < \epsilon_\theta$ at a probability of $p_j = \frac{e^{\epsilon_j}-1}{e^{\epsilon_\theta}-1}$. In SM_d, the server employs a DP mechanism (e.g., the Laplace Mechanism) to report an obfuscated result that achieves ϵ_θ-DP.

SM introduces two types of errors: *sampling error* and *noise error*. At each time slot t, given a privacy budget threshold ϵ_θ, the data reporting error is $err(\epsilon_\theta) = err_s(\epsilon_\theta) + err_{dp}(\epsilon_\theta)$. Here, $err_s(\epsilon_\theta)$ represents the *sampling error* from sampling users with privacy budgets below ϵ_θ, while $err_{dp}(\epsilon_\theta)$ represents the *noise error* from adding noise to achieve ϵ_θ-DP. Next, we introduce these sampling and noise errors in detail.

Definition 6. (Sampling Error [20]). Given a privacy budget threshold ϵ_θ and m kinds of privacy budget requirements $\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots, \tilde{\epsilon}_m$ from n users with $\tilde{\epsilon}_i < \tilde{\epsilon}_j$ for $i < j$ and $i, j \in [m]$ where $\tilde{\epsilon}_i$ is declared by n_i users ($\sum_{i=1}^{m} n_i = n$), the sampling error $err_s(\epsilon_\theta)$ is defined as

$$err_s(\epsilon_\theta) = Var(count(\boldsymbol{r}_t)) + bias(\boldsymbol{r}_t)^2 = \sum_{\tilde{\epsilon}_i < \epsilon_\theta} n_i p_i (1-p_i) + \left(\sum_{\tilde{\epsilon}_i < \epsilon_\theta} n_i (1-p_i) \right)^2, \tag{1}$$

where $p_i = \frac{e^{\tilde{\epsilon}_i}-1}{e^{\epsilon_\theta}-1}$.

Definition 7. (Noise Error). The noise error $err_{dp}(\epsilon_\theta)$ is defined as the error of the Laplace mechanism, namely,

$$err_{dp}(\epsilon_\theta) = \frac{2}{\epsilon_\theta^2}. \tag{2}$$

Various metrics exist to measure the errors of Laplace mechanisms for noise error, including variance [20, 29], scale [14, 22], and (α, β)-usefulness [5, 14]. In this work, we employ variance as our metric [20].

Based on Equations (1) and (2), we can observe that err_s depends on n_i, $\tilde{\epsilon}_i$ and ϵ_θ, and is independent of $\boldsymbol{r}_t$. Similarly, err_{dp} depends on ϵ_θ, and is independent of $\boldsymbol{r}_t$.

4.2 Optimal Budget Selection

Given the privacy budget requirements $(\epsilon_{1,t}, \epsilon_{2,t}, \ldots, \epsilon_{n,t})$ of n users, we can determine the frequency of each privacy budget requirement and select the optimal ϵ_θ that minimizes the data reporting error err. This process is detailed in Algorithm 1.

Taking n privacy budgets as input, the Optimal Budget Selection (OBS) algorithm counts the different privacy budgets. Assume there are $\tilde{n}$ distinct privacy budgets, with n_k users requiring $\tilde{\epsilon}_k$ for $k \in [\tilde{n}]$. Let $\tilde{\epsilon}$ be the set of different privacy budget and N be their corresponding frequencies (Lines 1-2). Then, OBS finds the minimum reporting error err_{min} (lines 4-8). Specifically, it iterates

Algorithm 1: Optimal Budget Selection (OBS)

Input: personalized privacy budget set $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$
Output: $\epsilon_{opt}, err_{min}$

```
Set ε̃ = (ε̃_1, ε̃_2, ..., ε̃_ñ) as the set of different ε ∈ ε;
Set N = (n_1, n_2, ..., n_ñ) as the corresponding frequency of ε̃_k ∈ ε̃;
Initialize err_min as the upper bound of error value;
for ε̃_k ∈ ε̃ do
    err = err_s(ε̃_k) + err_dp(ε̃_k);
    if err < err_min then
        err_min = err;
        ε_opt as ε̃_k;
return ε_opt, err_min
```

over all $\tilde{\epsilon}_k \in \tilde{\boldsymbol{\epsilon}}$ and selects the value $\tilde{\epsilon}_k$ with the smallest reporing error $err = err_s(\tilde{\epsilon}_k) + err_{dp}(\tilde{\epsilon}_k)$.

Example 2 (Running Example of the OBS Algorithm). *Suppose we have* 10 *privacy budgets as input:* $\boldsymbol{\epsilon} = (0.1, 0.4, 0.4, 0.1, 0.4, 0.4, 0.8, 0.8, 0.8, 0.4)$. *OBS first determines* $\tilde{\boldsymbol{\epsilon}} = (0.1, 0.4, 0.8)$, $\tilde{n} = |\tilde{\boldsymbol{\epsilon}}| = 3$, *and* $N = (2, 5, 3)$. *Based on these statistics, OBS iterates through the* 3 *privacy budgets in* $\tilde{\boldsymbol{\epsilon}}$ *and calculates the errors:* $err_1 = 0 + \frac{2}{0.1^2} = 200$, $err_2 = 2 \times \frac{e^{0.1}-1}{e^{0.4}-1} \times (1 - \frac{e^{0.1}-1}{e^{0.4}-1}) + (2 \times (1 - \frac{e^{0.1}-1}{e^{0.4}-1}))^2 + \frac{2}{0.4^2} = 15.31$ *and* $err_3 = 2 \times \frac{e^{0.1}-1}{e^{0.8}-1} \times (1 - \frac{e^{0.1}-1}{e^{0.8}-1}) + 5 \times \frac{e^{0.4}-1}{e^{0.8}-1} \times (1 - \frac{e^{0.4}-1}{e^{0.8}-1}) + (2 \times (1 - \frac{e^{0.1}-1}{e^{0.8}-1}) + 5 \times (1 - \frac{e^{0.4}-1}{e^{0.8}-1}))^2 + \frac{2}{0.8^2} = 89.74$. *Finally, OBS returns* 0.4 *with the minimal error* 15.31.

4.3 Personalized Window Size Mechanism

Budget division [22, 29] is a traditional framework for publishing private stream data under w-event privacy. It comprises two basic methods, namely *Uniform* and *Sampling* and two adaptive methods, namely *Budget Distribution* (BD) and *Budget Absorption* (BA). The adaptive methods leverage the stream's variation tendency, resulting in more accurate obfuscated estimations.

In this subsection, we extend the adaptive budget division framework to a personalized context and introduce our Personalized Window Size Mechanism (PWSM). Based on PWSM, we propose two methods: Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA).

In real applications, users must specify their privacy budgets and window sizes. System administrators first define a discretized privacy budget range (e.g., {0.1, 0.5, 0.9}) and a window size range (e.g., {40, 80, 120}). Then, they map ascending privacy budget values to descending privacy budget levels (e.g., High, Medium, Low) and ascending window size values to ascending window size levels (e.g., Small, Medium, Large). Users can then select both a privacy budget level and a window size level based on their needs and past experience. After users submit these selections, the server converts them into the corresponding values.

As shown in Algorithm 2, the PWSM algorithm takes three inputs: the historical estimation His, personalized privacy budget $\boldsymbol{\epsilon}$, and personalized window size set $\boldsymbol{w}$. Both $\boldsymbol{\epsilon}$ and $\boldsymbol{w}$ are fixed values collected from all users during system initialization. PWSM first calculates all users' privacy budget resources $\boldsymbol{\epsilon}_t$ at the current time slot t to satisfy $(\boldsymbol{w},\boldsymbol{\epsilon})$-EPDP (line 1). It then divides $\boldsymbol{\epsilon}_t$ into two parts: $\boldsymbol{\epsilon}_t^{(1)}$ and $\boldsymbol{\epsilon}_t^{(2)}$ (line 2). Using $\boldsymbol{\epsilon}_t^{(1)}$, PWSM calculates

Algorithm 2: PWSM

Input: historical estimation His, privacy requirement $(\boldsymbol{w}, \boldsymbol{\epsilon})$
Output: $\boldsymbol{r}$

```
1 Get the current privacy budgets ε_t of all users as ε and w;
2 Divide ε_t into two parts ε_t^(1) and ε_t^(2) satisfying ε_t = ε_t^(1) + ε_t^(2);
3 Calculate dissimilarity dis between current estimation and the last
    estimation by SM(ε_t^(1));
4 Calculate the reporting error err of current estimation by
    OBS(ε_t^(2));
5 if dis > √err then
6     Calculate current estimation r by SM(ε_t^(2));
7 else
8     Set current estimation r as the last reporting value;
9 return r;
```

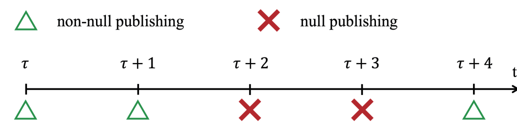


Figure 2: A non-null publishing example.

the dissimilarity dis between the current count value and the last published one by invoking the SM method [20] (line 3). Next, it sets the change threshold as the reporting error err calculated with $\boldsymbol{\epsilon}_t^{(2)}$ (line 4). Finally, PWSM adaptively decides whether to publish a new obfuscated estimation or skip (i.e., use the last published one to approximate) by comparing dis to $\sqrt{err}$ (lines 5-9).

To determine whether to publish a new obfuscated estimation or skip, we need to introduce a judgment measure called the *personalized private dissimilarity measure.*

Personalized Private Dissimilarity Measure. The personalized dissimilarity measure dis^* is defined as the absolute error between the true statistic $\tilde{\boldsymbol{c}}_t$ under SM_s (i.e., the *sample* step of SM) at current time slot t and the last publishing $\boldsymbol{r}_l$, namely,

$$dis^* = \frac{1}{d}\sum_{k=1}^{d} |\tilde{\boldsymbol{c}}_t[k] - \boldsymbol{r}_l[k]|.$$

Our goal is to privately obtain the personalized dissimilarity dis^* using the optimal privacy budget ϵ_{opt} calculated through *OBS* algorithm. The personalized private dissimilarity measure dis is defined as:

$$dis = dis^* + Lap\left(\frac{1}{d \cdot \epsilon_{opt}}\right),$$

where Lap represents the Laplace mechanism.

4.4 Personalized Budget Distribution and Personalized Budget Absorption

We first introduce some notations to further clarify PWSM in Algorithm 2, and then propose two solutions to implement PWSM in different scenarios.

Basic notations. For a sequence of publications $(\boldsymbol{r}_1, \boldsymbol{r}_2, ..., \boldsymbol{r}_t)$ of length t, we define a *null publishing* as an approximation value and *non-null publishing* as a new value. For any time slot $2 \le \tau \le t$, we refer to $\boldsymbol{r}_{\tau-1}$ as the last reporting value (or last publishing) of time slot τ. In the sequence $(\boldsymbol{r}_1, \boldsymbol{r}_2, ..., \boldsymbol{r}_\tau)$, we define the most recent non-null publishing $\boldsymbol{r}_l$ where $l < \tau$ as the last non-null publishing. For example in Figure 2, the publications at time slots $\tau, \tau + 1, \tau + 4$

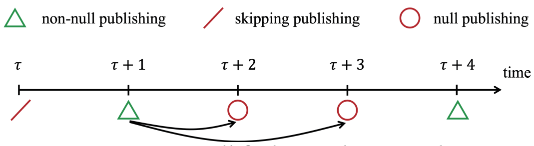


Figure 3: A nullified time slot example.

Algorithm 3: Dissimilarity Calculation (DC)

Input: D_t, current personalized privacy budget list $\boldsymbol{\epsilon}_t$, historical data publication $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$
Output: $\boldsymbol{r}_t$

1 ϵ_{opt}= OBS($\boldsymbol{\epsilon}_t$) ;
2 $\tilde{D}_t = SM_s(D_t, \boldsymbol{\epsilon}_t, \epsilon_{opt})$;
3 $\tilde{\boldsymbol{c}}_t = Q(\tilde{D}_t)$;
4 Get the last non-null publishing $\boldsymbol{r}_l$ from $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$;
5 **return** $dis = \frac{1}{d}\sum_{j=1}^{d} |\tilde{c}_t[j] - \boldsymbol{r}_l[j]| + Lap(1/(d \cdot \epsilon_{opt}))$;

Algorithm 4: Personalized Budget Distribution

Input: D_t, privacy requirement $(\boldsymbol{w}, \boldsymbol{\epsilon})$, historical data publication $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$
Output: $\boldsymbol{r}_t$

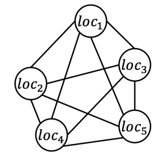
1 Get the current window average budget $\bar{\epsilon}_i = \epsilon_i / w_i$ for each $i \in [n]$;
2 $\boldsymbol{\epsilon}_t^{(1)} = (\bar{\epsilon}_1/2, \bar{\epsilon}_2/2, \ldots, \bar{\epsilon}_n/2)$;
3 Get dissimilarity dis by DC($D_t, \boldsymbol{\epsilon}_t^{(1)}, \boldsymbol{r}_1, ..., \boldsymbol{r}_{t-1}$) in Algorithm 3;
4 $\epsilon_{rm,i} = \epsilon_i/2 - \sum_{k=t-w_i+1}^{t-1} \epsilon_{i,k}^{(2)}$;
5 $\boldsymbol{\epsilon}_t^{(2)} = (\epsilon_{rm,1}/2, \epsilon_{rm,2}/2, \ldots, \epsilon_{rm,n}/2)$;
6 $\epsilon_{opt}^{(2)}, err_{opt}^{(2)}$ = OBS($\boldsymbol{\epsilon}_t^{(2)}$);
7 **if** $dis > \sqrt{err_{opt}^{(2)}}$ **then**
8 $\quad \tilde{D}_t^{(2)} = SM_s(D_t, \boldsymbol{\epsilon}_t^{(2)}, \epsilon_{opt}^{(2)})$;
9 $\quad \tilde{\boldsymbol{c}}_t^{(2)} = Q(\tilde{D}_t^{(2)})$;
10 $\quad$ **return** $\boldsymbol{r}_t = SM_d(\tilde{\boldsymbol{c}}_t^{(2)}, \epsilon_{opt}^{(2)})$;
11 **else**
12 $\quad$ **return** $\boldsymbol{r}_t = \boldsymbol{r}_{t-1}$;

are non-null publishing, while those at $\tau + 2$ and $\tau + 3$ are null publishing.

Given a privacy budget ϵ with window size w, the budget share $\bar{\epsilon} = \epsilon/w$ represents the smallest unit of privacy budget. The target is to maintain the total budget below ϵ within any w window while maximizing utility. Assume publishing new obfuscated data costs x budget shares ($x > 1$), the following $x - 1$ time slots use approximated values from their last reporting values. We refer to these $x-1$ time slots as nullified time slots. For example, in Figure 3, with $\epsilon = 4$ and $w = 4$, $\bar{\epsilon} = 1$. When time slot $\tau + 1$ uses 3 shares, the time slots $\tau + 2$ and $\tau + 3$ become nullified.

Personalized Budget Distribution (PBD). As shown in Algorithm 4, PBD inputs the current user data D_t, all users' privacy requirements, and historical data publication. The privacy budget ϵ_i of each user u_i is divided into two parts: 1) calculate the dissimilarity between the current data distribution and the last published obfuscated data distribution (denoted as Part$_{DC}$) (Lines 2-3); 2) calculate the new obfuscated publication at the current time slot (denoted as Part$_{NOP}$) (Lines 4-6 and Lines 8-10).

	u_1	u_2	u_3	…
$\boldsymbol{\epsilon}$	ϵ_1	ϵ_2	ϵ_3	…
$\boldsymbol{w}$	4	2	3	…



	1	2	3	4	5
u_1	loc_2	loc_1	loc_1	loc_3	loc_2
u_2	loc_1	loc_1	loc_3	loc_3	loc_4
u_3	loc_5	loc_4	loc_4	loc_2	loc_4
…	…	…	…	…	…

Figure 4: An Information example for PBD.

In Part$_{DC}$, we allocate half of the average privacy budget per time slot for dissimilarity calculation (i.e., $\frac{\epsilon_i}{2w_i}$ for u_i). The process then calls the Dissimilarity Calculation (Algorithm 3) to determine the dissimilarity. Within Algorithm 3, the OBS algorithm selects the optimal budget threshold ϵ_{opt}. Finally, it uses the SM [20] to compute the dissimilarity dis (lines 2-5).

In Part$_{NOP}$, we first calculate the remaining privacy budget $\epsilon_{rm,i}$ for each u_i. We then set the publication privacy budget for each u_i to half of $\epsilon_{rm,i}$. Similar to dissimilarity calculation, we use the OBS algorithm to determine the optimal privacy budget $\epsilon_{opt}^{(2)}$ and its corresponding error $err_{opt}^{(2)}$. At this point, we have obtained two measurements: the dissimilarity dis and the square root of error $\sqrt{err_{opt}^{(2)}}$. We compare these two measurements to determine whether to publish a new obfuscated statistic result or approximate the current result with the last publication. If the dis is greater than $\sqrt{err_{opt}^{(2)}}$, it indicates that the difference between the current data and the last published data exceeds the error of noise, then we republish a new obfuscated statistic result. Otherwise, we take the last published result instead.

We illustrate the process of Personalized Budget Distribution with an example as follows:

Example 3. *Suppose there are n users distributed across 5 locations, forming a complete graph. Figure 4 illustrates the privacy budget requirements, window size requirements and locations for the first three users across time slots 1 to 5. Figure 5 demonstrates the estimation process of PBD. The total privacy budget for each user u_i is evenly split into two parts, each containing $\epsilon_i/2$. The first part is allocated for dissimilarity calculation, while the second is for publication noise calculation. For instance, ϵ_1 is divided into $\boldsymbol{\epsilon}_1^{(1)}(u_1) = \epsilon_1/2$ and $\boldsymbol{\epsilon}_1^{(2)}(u_1) = \epsilon_1/2$. We compute the privacy budget usage $\epsilon_{i,t}^{(1)}$ for dissimilarity and $\epsilon_{i,t}^{(2)}$ for noise statistic publication for each user at each time slot. These values are recorded in an $n \times 2$ matrix at each time slot in Figure 5. Using u_1 as an example, $\epsilon_{1,t}^{(1)} = \boldsymbol{\epsilon}_1^{(1)}(u_1)/w_1 = \epsilon_1/8$. At time slot 1, $\epsilon_{1,1}^{(2)} = \boldsymbol{\epsilon}_1^{(2)}(u_1)/2 = \epsilon_1/4$. The algorithm calculates the dissimilarity dis at time slot 1 using all $\epsilon_{i,1}^{(1)}$, and the error $err_{opt}^{(2)}$ using all $\epsilon_{i,1}^{(2)}$. Assume dis $> \sqrt{err_{opt}^{(2)}}$, then a new obfuscated statistic $\boldsymbol{r}_1$ is published at time slot 1. At time slot 2, assume dis $\leq \sqrt{err_{opt}^{(2)}}$, then $\epsilon_{i,2}^{(2)}$ is not used to publish a new obfuscated*

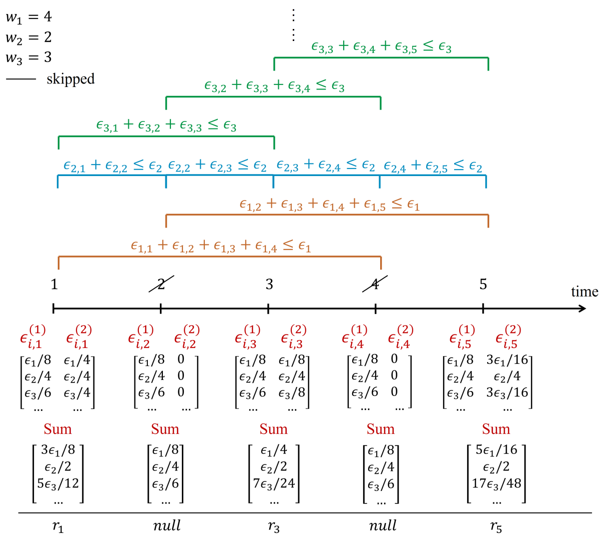


Figure 5: A process example for PBD.

statistic result, and its usage is set to zeros for all users. At time slot 3, $\epsilon_{1,3}^{(2)} = (\epsilon_1/2 - \epsilon_{1,1}^{(2)})/2 = \epsilon_1/8$. The vector below each matrix in Figure 5 represents the total privacy budget used at the current time slot for each user. For example, at time slot 1, the total privacy budget usage for u_1 is $\epsilon_{1,1}^{(1)} + \epsilon_{1,1}^{(2)} = 3\epsilon_1/8$.

Personalized Budget Absorption (PBA). Algorithm 5 outlines the process of PBA. The dissimilarity calculation (Part_{DC}) in PBA is identical to that of PBD. However, PBA and PBD differ significantly in their strategies on allocating the publication privacy budget.

For Part_{NOP} in PBA, we allocate an average privacy budget of $\frac{\epsilon_i}{2w_i}$ (one share) for each u_i at each time slot t. A publication at time slot t can use more than one share by borrowing from its successor time slots. The variable $t_{i,N}$ in Line 5 represents the number of successor time slots occupied by the last publication. We calculate the maximal $\tilde{t}_N$ of all $t_{i,N}$ and determine whether the current time has been occupied ($t - l \le \tilde{t}_N$). If so, we approximate the publication using the last published result. Otherwise, we calculate the remaining budget shares from the precursor time slots (i.e., $t_{A,i}$ in Line 10) and set the current publication budget as the total absorbed shares (Line 11). The subsequent steps follow the same process as outlined in Algorithm 4.

Example 4. *We continue use the demonstration case shown in Figure 4. Figure 6 illustrates the estimation process of PBA. The dissimilarity calculation process in PBA is identical to that in Example 3. For Part_{NOP}, at time slot 1, with no budget to absorb, all users utilize one share (i.e., $\epsilon_i/(2w_i)$) to publish a new obfuscated statistic result. Assume time slot 2 is skipped (i.e., $dis \le \sqrt{err_{opt}^{(2)}}$). At time slot 3, $t_{1,N} = t_{2,N} = t_{3,N} = 0$. Thus, the nullified bound $\tilde{t}_N$ is 0. Since $t - l = 3 - 1 = 2 > \tilde{t}_N$, a new obfuscated statistic result is reported. The publication budget set is calculated as $\boldsymbol{\epsilon}_3^{(2)} = (\epsilon_1/4, \epsilon_2/2, \epsilon_3/3, \ldots)$. At time slot 4, $t_{1,N} = t_{2,N} = t_{3,N} = 1$. As $t - l = 4 - 3 = 1 \le \tilde{t}_N$, no output is produced. At time slot 5, all $t_{i,N}$ remain 1, and $t - l = 5 - 3 = 2 > \tilde{t}_N$. The absorbed time slots $t_{A,i}$ all equal 1. The resulting publication budget set is $\boldsymbol{\epsilon}_5^{(2)} = (\epsilon_1/8, \epsilon_2/4, \epsilon_3/6, \ldots)$.*

Algorithm 5: Personalized Budget Absorption

Input: D_t, EPDP privacy requirement $(\boldsymbol{w}, \boldsymbol{\epsilon})$, historical data publication $(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1})$

Output: $\boldsymbol{r}_t$

1 Get the current window average budget $\bar{\epsilon}_i = \epsilon_i / w_i$ for each $i \in [n]$;

2 $\boldsymbol{\epsilon}_t^{(1)} = (\bar{\epsilon}_1/2, \bar{\epsilon}_2/2, \ldots, \bar{\epsilon}_n/2)$;

3 Get dissimilarity dis by DC($D_t, \boldsymbol{\epsilon}_t^{(1)}, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_{t-1}$) in Algorithm 3;

4 Initialize nullified time slots $t_{i,N}$ as 0;

5 Set $t_{i,N} = \frac{\epsilon_{i,l}^{(2)}}{\epsilon_i/(2w_i)} - 1$ for $i \in [n]$ if l exists where l is the last non-null publishing time slot;

6 Set nullified time slot bound $\tilde{t}_N = \max_{i \in [n]} t_{i,N}$;

7 **if** $t - l \le \tilde{t}_N$ **then**

8 **return** $\boldsymbol{r}_t = \boldsymbol{r}_{t-1}$;

9 **else**

10 Set absorbed time slots $t_{A,i} = \max(t - l - t_{i,N}, 0)$ for $i \in [n]$;

11 Set publication budget $\epsilon_{i,t}^{(2)} = \frac{\epsilon_i}{2w_i} \cdot \min(t_{A,i}, w_i)$ for $i \in [n]$;

12 $\boldsymbol{\epsilon}_t^{(2)} = \left(\epsilon_{1,t}^{(2)}, \epsilon_{2,t}^{(2)}, \ldots, \epsilon_{n,t}^{(2)}\right)$;

13 $\epsilon_{opt}^{(2)}, err_{opt}^{(2)} = \text{OBS}(\boldsymbol{\epsilon}_t^{(2)})$;

14 **if** $dis > \sqrt{err_{opt}^{(2)}}$ **then**

15   $\tilde{D}_t^{(2)} = SM_s(D_t, \boldsymbol{\epsilon}_t^{(2)}, \epsilon_{opt}^{(2)})$;

16   $\tilde{c}_t^{(2)} = Q(\tilde{D}_t^{(2)})$;

17   **return** $\boldsymbol{r}_t = SM_d(\tilde{c}_t^{(2)}, \epsilon_{opt}^{(2)})$;

18 **else**

19   **return** $\boldsymbol{r}_t = \boldsymbol{r}_{t-1}$;

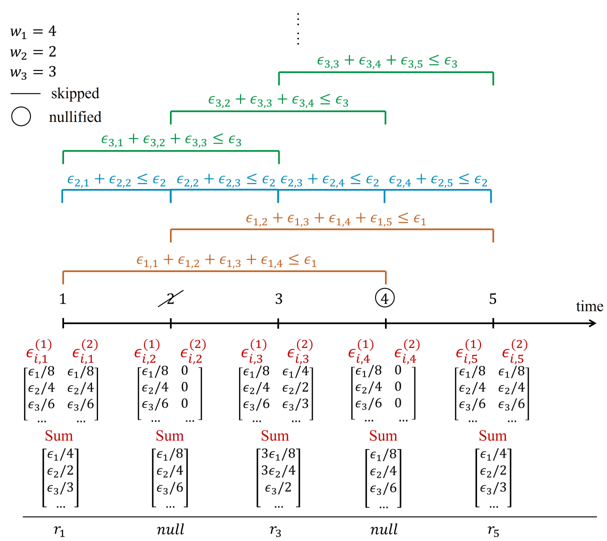


Figure 6: A process example for PBA.

4.5 Analyses

Time Cost Analysis. Let m be the number of distinct privacy requirements (w_i, ϵ_i), where $m \le n$. The time complexity of OBS is $O(m)$ for both PBD and PBA. The Sample Mechanism and Query operations each have a time complexity of $O(n)$. Thus, the time complexities of PBD and PBA both are $O(n)$.

Privacy Analysis. The privacy analysis for PBD and PBA:

Theorem 4.1. *PBD and PBA satisfy $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP.*

Proof. (1) PBD satisfies $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP.

In the process of Part_{DC}, for each user u_i, the dissimilarity budget at each time slot is $\epsilon_i/(2w_i)$. Then for each time slot t, we have $\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} = \epsilon_i/2$.

In Part_{NOP}, for each user u_i at time slot t, only half of the publication budget is used when publication occurs: $\epsilon_{i,t}^{(2)} = (\epsilon_i/2 - \sum_{k=\max(t-w_i+1,1)}^{t-1} \epsilon_{i,k}^{(2)})/2$. For any time slot $t \in [1, w_i]$, the summation publication budgets used for u_i is at most $\sum_{k=1}^{w_i} \epsilon_i/(2 \cdot 2^k) \leq (\epsilon_i/2) \cdot (1 - \frac{1}{2^{w_i}}) \leq \epsilon_i/2$. Suppose $\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \epsilon_i/2$ for $t = w_i + s$ (i.e., $\sum_{k=\max(s+1,1)}^{w_i+s} \epsilon_{i,k}^{(2)} \leq \epsilon_i/2$). Then for $t = w_i + s + 1$, we have:

$$\sum_{k=\max(s+2,1)}^{w_i+s+1} \epsilon_{i,k}^{(2)} = \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)} + \epsilon_{i,w_i+s+1}^{(2)}. \tag{3}$$

Since $\epsilon_{i,w_i+s+1}^{(2)}$ is at most half of the remaining publication budget at time slot $w_i + s$:

$$\epsilon_{i,w_i+s+1}^{(2)} \leq (\epsilon_i/2 - \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2. \tag{4}$$

According to Equations (3) and (4), we have:

$$\begin{aligned}\sum_{k=\max(s+2,1)}^{w_i+s+1} \epsilon_{i,k}^{(2)} &\leq \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)} + (\epsilon_i/2 - \sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2 \\ &= \epsilon_i/4 + (\sum_{k=\max(s+2,1)}^{w_i+s} \epsilon_{i,k}^{(2)})/2 \\ &\leq \epsilon_i/4 + \epsilon_i/4 \\ &= \epsilon_i/2.\end{aligned}$$

Therefore, for any $t \geq 1$, we have:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \epsilon_i/2.$$

According to the Composition Theorems [14], we have:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k} = \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} + \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \epsilon_i.$$

For any user u_i and any two w_i-neighboring stream prefixes S_t and S'_t (i.e., $S_t \sim_{w_i} S'_t$), let t_s be the earliest time slot where $S_t[t_s] \neq S'_t[t_s]$ and t_e be the latest time slot where $S_t[t_e] \neq S'_t[t_e]$. Then we have $t_e - t_s + 1 \leq w_i$. Denoting the output of our PBD as $PBD(S_t[t]) = o_t \in \mathcal{O}$, for any $O \subseteq \mathcal{O}$, we have:

$$\begin{aligned}\frac{\Pr[PBD(S_t)] \in O}{\Pr[PBD(S'_t)] \in O} &\leq \Pi_{k=t_s}^{t_e} \frac{\Pr[PBD(S_t[k]) = o_k]}{\Pr[PBD(S'_t[k]) = o_k]} \\ &\leq e^{\sum_{k=t_s}^{t_e} \epsilon_{i,k}} \leq e^{\sum_{k=\max(t_e-w_i+1,1)}^{t_e} \epsilon_{i,k}} \leq e^{\epsilon_i}.\end{aligned}$$

Therefore, PBD satisfies $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP where $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ and $\boldsymbol{\epsilon} = ((u_1, \epsilon_1), (u_2, \epsilon_2), \ldots, (u_n, \epsilon_n))$.

(2) PBA satisfies $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP.

The Part_{DC} in PBA is identical to that that in PBD. Consequently, for each time slot t, we have:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} = \epsilon_i/2. \tag{5}$$

In Part_{NOP}, for any user u_i and any window of size w_i, there are s_i publication time slots in the window. We denote these publication time slots as $(k_1, k_2, \ldots, k_{s_i})$. For any publication time slot k_j ($j \in$

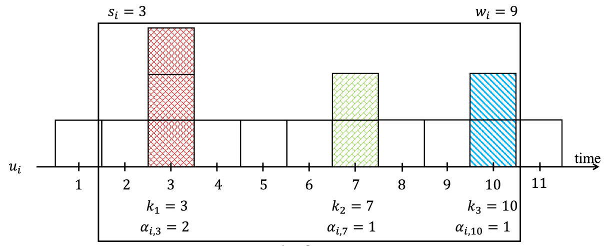


Figure 7: An example for parameters in PBA.

$[s_i]$), the quantity of its absorbing unused budgets is denoted as α_{i,k_j}. Figure 7 illustrates an example where $s_i = 3$ and $w_i = 9$.

Based on Algorithm 5, we have:

$$w_i \geq \sum_{j=1}^{s_i} (1 + 2\alpha_{i,k_j}) - \alpha_{i,k_1} - \alpha_{i,k_{s_i}}.$$

Then, for the total publication budgets used in any window, we have

$$\begin{aligned}\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} &\leq \frac{\epsilon_i}{2w_i} \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j}) \\ &\leq \frac{\epsilon_i \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j})}{2\sum_{j=1}^{s_i} (1 + 2\alpha_{i,k_j}) - 2\alpha_{i,k_1} - 2\alpha_{i,k_{s_i}}} \\ &= \frac{\epsilon_i \cdot \sum_{j=1}^{s_i} (1 + \alpha_{i,k_j})}{2\sum_{j=1}^{s_i} (1 + \alpha_{i,k_j}) + 2\sum_{j=2}^{s_i-1} \alpha_{i,k_j}} \\ &\leq \epsilon_i/2.\end{aligned} \tag{6}$$

Based on Equations (5) and (6), and applying the Composition Theorems [14], we obtain:

$$\sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k} = \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(1)} + \sum_{k=\max(t-w_i+1,1)}^{t} \epsilon_{i,k}^{(2)} \leq \epsilon_i.$$

The subsequent proof process follows the same steps as in PBD. Ultimately, we demonstrate that PBA also satisfies $(\boldsymbol{w}, \boldsymbol{\epsilon})$-EPDP. □

Utility Analysis. For each user u_i in PBD and PBA, we define w_L as the smallest window size among all users. For each u_i, given (w_i, ϵ_i), let $\epsilon_L = \min_{i\in[n]} \frac{\epsilon_i}{w_i}$ and $\epsilon_R = \max_{i\in[n]} \frac{\epsilon_i}{w_i}$ represent the minimum and maximum values of $\frac{\epsilon_i}{w_i}$, respectively. Let n_A be the number of occurrences of ϵ_R across all users.

We make the following assumptions: At most $\tilde{s} \leq w_L$ publications occur at time slots $q_1, q_2, \ldots, q_{\tilde{s}}$ in the window of size w_L, with no budget absorption from past time slots outside the window. Additionally, for each user, each publication approximates the same number of skipped or nullified publications.

We first present a crucial lemma, followed by two theorems that bound the average errors of PBD and PBA, respectively.

Lemma 4.1. *Given m distinct privacy budget-quantity pairs $P = \{(\epsilon_j, n_j) | j \in [m], \sum_{j\in[m]} n_j = n\}$ where pair (ϵ_j, n_j) indicates that ϵ_j appears n_j times in the user privacy requirement, and a query with sensitivity I, the error upper bound $\widetilde{err}_O(P)$ of the SM process with privacy budget chosen from OBS is:*

$$\min\left(\frac{2I^2}{\min_j \epsilon_j^2}, (n - n_M)(n - n_M + \frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2}\right),$$

where $n_M = n_k$ with $k = \arg\max_{j\in[m]} \epsilon_j$.

Proof. Let M_L be the SM with privacy budget chosen as $\min_j \epsilon_j$. According to the SM process, all budget types will be selected. In this case, the sampling error err_s is 0 and the noise error err_{dp}

is $2 \cdot (\frac{I}{\min_j \epsilon_j})^2 = \frac{2I^2}{\min_j \epsilon_j^2}$. Thus, the total error of M_L is $err_{M_L} = \frac{2I^2}{\min_j \epsilon_j^2}$. Let M_R be the SM with privacy budget chosen as $\max_j \epsilon_j$. In this case, $(m-1)$ types of privacy budget are chosen with probability $p_k = \frac{e^{\epsilon_k}-1}{e^{\max_j \epsilon_j}-1}$ less than 1 ($k \in [m]$). For the sampling error, we have:

$$\begin{aligned} err_s &= \sum_{\epsilon_k < \max_j \epsilon_j} n_k p_k (1-p_k) + \left(\sum_{\epsilon_k < \max_j \epsilon_j} n_k (1-p_k) \right)^2 \\ &< \sum_{\epsilon_k < \max_j \epsilon_j} n_k \left(\frac{p_k + 1 - p_k}{2} \right)^2 + \left(\sum_{\epsilon_k < \max_j \epsilon_j} n_k \right)^2 \\ &= (n - n_M)(n - n_M + \frac{1}{4}). \end{aligned}$$

The noise error err_{dp} in this case is $2 \cdot (\frac{I}{\max_j \epsilon_j})^2 = \frac{2I^2}{\max_j \epsilon_j^2}$. Thus, the total error of M_R is $err_{M_R} = (n-n_M)(n-n_M+\frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2}$. According to the OBS process, we have $\widetilde{err}_O(P) \leq err_{M_L}$ and $\widetilde{err}_O(P) \leq err_{M_R}$. Therefore,

$$\widetilde{err}_O(P) \leq \min(err_{M_L}, err_{M_R}) = \min\left(\frac{2I^2}{\min_j \epsilon_j^2}, (n-n_M)(n-n_M+\frac{1}{4}) + \frac{2I^2}{\max_j \epsilon_j^2} \right).$$

□

For PBD, we present Theorem 4.2 as follows.

Theorem 4.2. *The average error per time slot in PBD is at most* $\min\left(\frac{8}{d^2\epsilon_L}, Z + \frac{8}{d^2\epsilon_R}\right) + \min\left(\frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_L}, Z + \frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_R}\right)$ *where* $Z = (n-n_A)(n-n_A+\frac{1}{4})$*, if at most* $\tilde{s}$ *publications occur in any window with size* w_L*.*

Proof. Given a privacy budget-quantity pair set P, let $EOPT(P)$ be the optimal privacy budget chosen from OBS. Given a positive number β, we define $\beta \cdot P = \{(\beta \cdot \epsilon_j, n_j) | (\epsilon_j, n_j) \in P\}$. For each user u_i with privacy requirement pair (w_i, ϵ_i), we calculate their average budget per window as $\frac{\epsilon_i}{w_i}$. We denote the set of all average budgets as $\overline{\epsilon} = \{\frac{\epsilon_i}{w_i} | i \in [n]\}$. We then construct the privacy budget-quantity pair set of each type of average budget as $P_A = \{(\epsilon_j, n_j) | \epsilon_j \in \overline{\epsilon}\}$. Let $Z = (n-n_A)(n-n_A+\frac{1}{4})$ be the sampling error upper bound, where n_A is the quantity of $\max_{i\in[n]} \frac{\epsilon_i}{w_i}$ in $\overline{\epsilon}$.

When Part$_{DC}$ is not private, the error stems from Part$_{NOP}$. In Part$_{NOP}$, errors arise from both publications and approximations. According to the Part$_{NOP}$, an approximation error does not exceed the publication error at the most recent publication time slot. For the average error $\overline{err}_{NOP}$ of all time slots within the window of size w_L, based on the PBD process, we have:

$$\begin{aligned} \overline{err}_{NOP} &= \frac{1}{w_L} \sum_{k\in[\tilde{s}]} \frac{w_L}{\tilde{s}} \cdot \widetilde{err}_O\left(\frac{1}{2^{k+1}} P_A\right) \\ &< \frac{1}{\tilde{s}} \sum_{k\in[\tilde{s}]} \min\left(\frac{2}{(\frac{\epsilon_L}{2^{k+1}})^2}, Z + \frac{2}{(\frac{\epsilon_R}{2^{k+1}})^2} \right) \\ &< \frac{1}{\tilde{s}} \min\left(\sum_{k\in[\tilde{s}]} \frac{8\cdot 4^k}{\epsilon_L^2}, \tilde{s}\cdot Z + \sum_{k\in[\tilde{s}]} \frac{8\cdot 4^k}{\epsilon_R^2} \right) \\ &= \min\left(\frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_L^2}, Z + \frac{32\cdot(4^{\tilde{s}}-1)}{3\tilde{s}\epsilon_R^2} \right). \end{aligned} \tag{7}$$

When Part$_{DC}$ is private, the error from Part$_{DC}$ can lead to two scenarios: (1) falsely skipping a publication or (2) falsely performs a publication. Both cases are bounded by the error in Part$_{DC}$. In Part$_{DC}$, we execute the SM with OBS. The sensitivity of dis is $1/d$. For the average error $\overline{err}_{DC}$ of each time slot in window size w_L, according to Lemma 4.1, we have:

$$\begin{aligned} \overline{err}_{DC} &< \min\left(\frac{2}{d^2 \min_{i\in[n]} (\frac{\epsilon_i}{2w_i})^2}, Z + \frac{2}{d^2 \max_{i\in[n]} (\frac{\epsilon_i}{2w_i})^2} \right) \\ &= \min\left(\frac{8}{d^2\epsilon_L^2}, Z + \frac{8}{d^2\epsilon_R^2} \right). \end{aligned} \tag{8}$$

Based on Equation (8) and (7), we can get the average error upper bound as $\overline{err}_{DC} + \overline{err}_{NOP}$.

□

PBD achieves low error when the number of publications $\tilde{s}$ per window is small. However, the error increases exponentially with $\tilde{s}$. Additionally, the error in Part$_{DC}$ (the first part of the error upper bound in PBD) rises as w_L increases, however, it diminishes as d increases. This is because a large d reduces sensitivity leading to smaller noise error.

For PBA, assume α skipped publications occur before a publication. Let $\epsilon_{\tilde{L}}$ and $\epsilon_{\tilde{R}}$ be the minimum and maximum publication privacy budget among all users at time slots $t = w_L$ and $t = (\alpha+1)$, respectively. According to the PBA process, there will be α nullified publications after the publication. These nullified publications are filled by the last time slot's publication without comparison. Consequently, the nullified publication error depends on the data distribution at nullified time slots. We denote the average error of each nullified publication in PBA as $\overline{err}_{nlf}$. For PBA, we have Theorem 4.3 as follows.

Theorem 4.3. *The average error per time slot in PBA is at most* $\min(\frac{8}{d^2\epsilon_L}, Z + \frac{8}{d^2\epsilon_R}) + \frac{1}{2\alpha+1}(\widetilde{err}_{NOP}^{(s,p)} + \alpha \cdot \overline{err}_{nlf})$ *where* $\widetilde{err}_{NOP}^{(s,p)}$ *is* $\min(\frac{2}{\epsilon_{\tilde{L}}^2} H_{\alpha+1}^2, (\alpha+1)Z + \frac{2}{\epsilon_{\tilde{R}}^2} H_{\alpha+1}^2)$ *when* $\alpha \leq w_L$ *and* $\min(\frac{2}{\epsilon_{\tilde{L}}^2} H_{w_L}^2, w_L Z + \frac{2}{\epsilon_{\tilde{R}}^2} H_{w_L}^2) + (\alpha - w_L + 1)\min(\frac{2}{\epsilon_{\tilde{L}}^2}, Z + \frac{2}{\epsilon_{\tilde{R}}^2})$ *when* $\alpha > w_L$ *and* $Z = (n-n_A)(n-n_A+\frac{1}{4})$ *and* H_x^2 *is the x-th square harmonic number, if there are* α *skipped publications occur in average before each publication.*

Proof. Similar to PBD, we first analyze the error of Part$_{NOP}$ in PBA by assuming Part$_{DC}$ is not private. We then add the error of Part$_{DC}$, which is identical to that in PBD, to obtain the final total error. When Part$_{DC}$ is not private, the error stems from Part$_{NOP}$. In Part$_{NOP}$, each publication corresponds to α skipped publications preceding it and α nullified publications succeeding it.

For each user u_i's skipped publication, the publication privacy budget lower bound doubles with each time slot increase until it reaches $\epsilon_i/2$ or a publication occurs. For example, in Figure 8, where $\alpha = 5$, the publication time slot is t_6. At time slot t_1, each u_i's publication budget lower bound is $\epsilon_i/(2w_i)$. Take u_1 as an example: it reaches $\epsilon_1/2$ at time slot t_4. The publication lower bound for u_1 remains at $\epsilon_1/2$ until time slot t_6. Let the publication budget lower bound set for all users at skipped time slots (spanning α time slot) be $\hat{\boldsymbol{\epsilon}} = \{\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \ldots, \boldsymbol{\epsilon}_\alpha\}$. Then, the error upper bound of each skipped publication is the error of publishing new data using $\boldsymbol{\epsilon}_k$ ($k \in [\alpha]$). For example in Figure 8, the error upper bound at t_3 is the error of publication a new obfuscated statistic result using $\{\frac{3\epsilon_1}{2}, \frac{\epsilon_2}{2}, \frac{3\epsilon_3}{16}, \frac{\epsilon_4}{4}\}$.

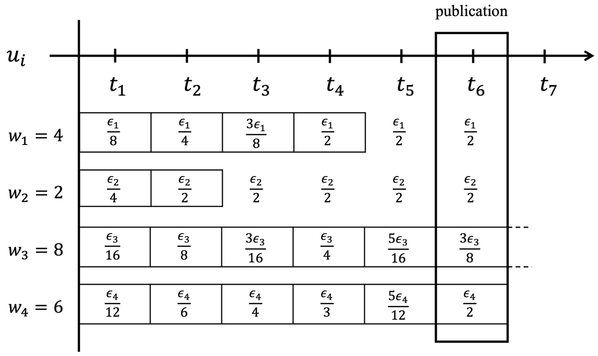


Figure 8: An example of the publication budget lower bound in PBA.

Let $Z = (n - n_A)(n - n_A + \frac{1}{4})$ be the sampling error upper bound, where n_A is the number of users with maximum value of $\frac{\epsilon_i}{w_i}$. We now consider two cases: $\alpha \leq w_L$ and $\alpha > w_L$.

(1) case 1: $\alpha \leq w_L$.

In this case, the publication budget lower bound doubles with each time slot increase. Let $err_{NOP}^{(sk)}(\alpha)$ and $err_{NOP}^{(pb)}$ be the total error upper bounds of the α skipped publications and the publication in Part$_{NOP}$, respectively. Let $err_{NOP}^{(s,p)}$ be the error of all skipped publications and the publication in Part$_{NOP}$. According to Lemma 4.1, we have

$$err_{NOP}^{(sk)}(\alpha) < \sum_{k\in[\alpha]} \min\left(\frac{2}{(k\epsilon_L)^2}, Z + \frac{2}{(k\epsilon_R)^2}\right) \leq \min\left(\frac{2}{\epsilon_L^2}H_\alpha^2, \alpha Z + \frac{2}{\epsilon_R^2}H_\alpha^2\right) \tag{9}$$

and

$$err_{NOP}^{(s,p)} < err_{NOP}^{(sk)}(\alpha) + err_{NOP}^{(pb)} = err_{NOP}^{(sk)}(\alpha+1) = \min\left(\frac{2}{\epsilon_L^2}H_{\alpha+1}^2, (\alpha+1)Z + \frac{2}{\epsilon_R^2}H_{\alpha+1}^2\right). \tag{10}$$

Thus, we derive the average error upper bound $\overline{err}_{NOP}$ of each time slot in Part$_{NOP}$ as

$$\overline{err}_{NOP} < \frac{1}{2\alpha+1}(\widetilde{err}_{NOP}^{(s,p)} + \alpha\cdot\overline{err}_{nlf}), \tag{11}$$

where $\widetilde{err}_{NOP}^{(s,p)}$ is the final value in Equation (10).

(2) case 2: $\alpha > w_L$.

In this case, we have

$$\begin{aligned} err_{NOP}^{(s,p)} &< err_{NOP}^{(sk)}(w_L) + \sum_{k=w_L+1}^{\alpha+1} \min\left(\frac{2}{\epsilon_L^2}, Z + \frac{2}{\epsilon_R^2}\right) \\ &= err_{NOP}^{(sk)}(w_L) + (\alpha - w_L + 1)\min\left(\frac{2}{\epsilon_L^2}, Z + \frac{2}{\epsilon_R^2}\right) \\ &< \min\left(\frac{2}{\epsilon_L^2}H_{w_L}^2, w_L Z + \frac{2}{\epsilon_R^2}H_{w_L}^2\right) + (\alpha - w_L + 1)\min\left(\frac{2}{\epsilon_L^2}, Z + \frac{2}{\epsilon_R^2}\right). \end{aligned} \tag{12}$$

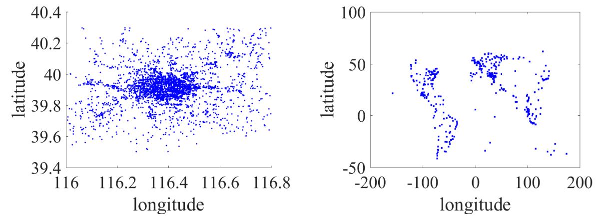
Therefore, we obtain the average error upper bound $\overline{err}_{NOP}$ for each time slot in Part$_{NOP}$ as

$$\overline{err}_{NOP} < \frac{1}{2\alpha+1}(\widetilde{err}_{NOP}^{(s,p)} + \alpha\cdot\overline{err}_{nlf}) \tag{13}$$

where $\widetilde{err}_{NOP}^{(s,p)}$ is the value derived in Equation (12).

When Part$_{DC}$ is private, its error is identical to that in PBD:

$$\overline{err}_{DC} < \min\left(\frac{8}{d^2\epsilon_L^2}, Z + \frac{8}{d^2\epsilon_R^2}\right). \tag{14}$$



(a) Taxi (b) Foursquare

Figure 9: Illustration of Real datasets.

Based on Equation (14), (11) and (13), we can derive the average error upper bound for each time slot in PBA as:

$$\min\left(\frac{8}{d^2\epsilon_L^2}, Z + \frac{8}{d^2\epsilon_R^2}\right) + \frac{1}{2\alpha+1}(\widetilde{err}_{NOP}^{(s,p)} + \alpha\cdot\overline{err}_{nlf}),$$

where $\widetilde{err}_{NOP}^{(s,p)}$ is the final result from Equation (10) when $\alpha \leq w_L$, and from Equation (12) when $\alpha > w_L$. □

5 EXPERIMENTS

5.1 Datasets

Real datasets. We use two real-world datasets, *Taxi* [37, 38] and *Foursquare* [35, 36], to evaluate the performance of our algorithms.

Taxi. It contains real-time trajectories of 10,357 taxis' in Beijing from February 2 to February 8, 2008. Each taxi has up to 154,699 records, where each record comprises *taxi id*, *data time*, *longitude* and *latitude*. For the spatial dimension, we first remove all duplicate records, then extract records with longitude between 116 and 116.8 and latitude between 39.5 and 40.3, resulting in 14,859,377 records. We denote this area ($[116, 116.8] \times [39.5, 40.3]$) as A_E. Figure 9(a) shows 50% of uniformly extracted trajectory points in A_E. We further divide A_E uniformly into a 10×10 grids, designating these 100 cells as the location space. For the time dimension, we sample records every minute and get 8,889 records.

Foursquare. It contains 33,278,683 Foursquare check-ins from 266,909 users, during April 2012 to September 2013. Each record consists of user id, venue id (place), and time. We convert the venue id to the country where the venue is located. After removing invalid records, we uniformly extract 5% of users' check-ins as shown in Figure 9(b). We set the publication time interval to 100 minutes, thus divide the chick-ins period into 7,649 time slots.

Synthetic datasets. We generate three binary stream datasets using different sequence models. We set the length of each binary stream as T and the number of users as N. For each stream, we first generate a probability sequence $(p_1, p_2, ..., p_T)$. At each time slot t, each user's real value is set to 1 with probability p_t and 0 otherwise. The probability function we use are as follows:

- TLNS function. In TLNS, $p_t = p_{t-1} + \mathcal{N}(0, Q)$, where $\mathcal{N}(0, Q)$ is Gaussian noise with standard variance $\sqrt{Q} = 0.0025$. We set $p_0 = 0.05$ as the initial value. If $p_t < 0$, we set $p_t = 0$; If $p_t > 1$, we set $p_t = 1$.
- Sin function. In Sin, $p_t = A\sin(\omega t) + h$, where $A = 0.05$, $\omega = 0.01$ and $h = 0.075$.
- Log function. In Log, $p_t = A/(1 + e^{-bt})$, where $A = 0.25$ and $b = 0.01$.

Table 3: Experimental settings.

Parameters	Values
static privacy budget ϵ	0.2, 0.4, **0.6**, 0.8, 1.0
static window size w	40, 80, **120**, 160, 200
personalized privacy budget ϵ_i	$\epsilon, \ldots, 0.8, 1.0$
personalized window size w_i	$40, 80, \ldots, w$
users' quantity ratio o	0.1, 0.3, **0.5**, 0.7, 0.9

5.2 Experiment Setup

We compare our PBD and PBA with two non-personalized methods: Budget Distribution (BD) and Budget Absorption (BA) [22]. We also compare against a simple personalized LDP method, Personalized LDP Budget Uniform (PLBU), which extends LDP Budget Uniform (LBU) [29] by replacing the inner CDP mechanism with an LDP mechanism.

Let ϵ and w be the privacy budget and window size in non-personalized static methods (BD and BA). For non-personalized static methods, we set the ϵ to vary from 0.2 to 1.0 and w to vary from 40 to 200. To make our PBD and PBA comparable with BD and BA, we set the lower bound of each user's privacy budget as ϵ and the upper bound of each user's window size as w in PBD and PBA to match the requirement of privacy level.

Given $\tilde{n}$ different privacy budgets $\tilde{\boldsymbol{\epsilon}} = \{\epsilon_1, ..., \epsilon_{\tilde{n}}\}$, let $N(\epsilon_i)$ be the count of budget value ϵ_i, and $N(\tilde{\boldsymbol{\epsilon}}) = \sum_{i=1}^{\tilde{n}} N(\epsilon_i)$ be the total count of all the budgets. For any $\epsilon_i \in \tilde{\boldsymbol{\epsilon}}$, we define the privacy budget ratio of ϵ_i as $\frac{N(\epsilon_i)}{N(\tilde{\boldsymbol{\epsilon}})}$. Similarly, we define the window size ratio of any w_i in different window sizes $\tilde{\boldsymbol{w}} = \{w_1, ..., w_{\tilde{n}}\}$ as $\frac{N(w_i)}{N(\tilde{\boldsymbol{w}})}$. We set the privacy domain as $\{0.5, 1.0\}$ and the window size domain as $\{10, 20\}$. We alter the ratio o of $\epsilon_i = 0.5$ and $w_i = 10$ from 0.1 to 0.9.

The parameters are shown in Table 3, where the default values are in bold font. We run the experiments on an Intel(R) Xeon(R) Silver 4210R CPU @ 2.4GHz with 128 RAM in Java. Each experiment is run 10 times, and we report the average result.

5.3 Measures

We evaluate the performance of different mechanisms based on their running time and data utility. We measure data utility as *Average Mean Relative Error* (*AMRE*) and *Average Jensen-Shannon Divergence* ($AJSD$, $\bar{D}_{JS}$). Let T represent the number of time slots and d denote the dimension of data.

AMRE is defined as the average value of Mean Relative Error (*MRE*), which is shown in Equation (15).

$$AMRE = \frac{1}{T}\sum_{\tau=1}^{T} MRE_\tau = \frac{1}{T}\sum_{\tau=1}^{T}\frac{1}{d}\|\boldsymbol{r}_\tau - \boldsymbol{c}_\tau\|_2^2. \tag{15}$$

$AJSD$ is defined as the average value of Jensen-Shannon Divergence (JSD, D_{JS}) [25], which is based on Kullback-Leibler Divergence [24], as shown in Equation (16).

$$\begin{aligned}\bar{D}_{JS}(\boldsymbol{r}\|\boldsymbol{c}) &= \frac{1}{T}\sum_{\tau=1}^{T} D_{JS}(\boldsymbol{r}\|\boldsymbol{c}) = \frac{1}{T}\sum_{\tau=1}^{T}\left(\frac{1}{2}D_{KL}(\boldsymbol{r}\|\boldsymbol{v}) + \frac{1}{2}D_{KL}(\boldsymbol{c}\|\boldsymbol{v})\right)\\ &= \frac{1}{2T}\sum_{\tau=1}^{T}\sum_{j=1}^{d}\left(\boldsymbol{r}_\tau(j)\log\left(\frac{\boldsymbol{r}_\tau(j)}{\boldsymbol{v}_\tau(j)}\right) + \boldsymbol{c}_\tau(j)\log\left(\frac{\boldsymbol{c}_\tau(j)}{\boldsymbol{v}_\tau(j)}\right)\right),\end{aligned} \tag{16}$$

where $\boldsymbol{v}$ represents the average distribution of $\boldsymbol{r}$ and $\boldsymbol{c}$, i.e., $\boldsymbol{v}(j) = \frac{1}{2}(\boldsymbol{r}(j) + \boldsymbol{c}(j))$. For time slot τ, $r_\tau(j)$ and $c_\tau(j)$ represent the j-th dimensional values in the obfuscated and original data, respectively.

5.4 Overall Utility Analysis

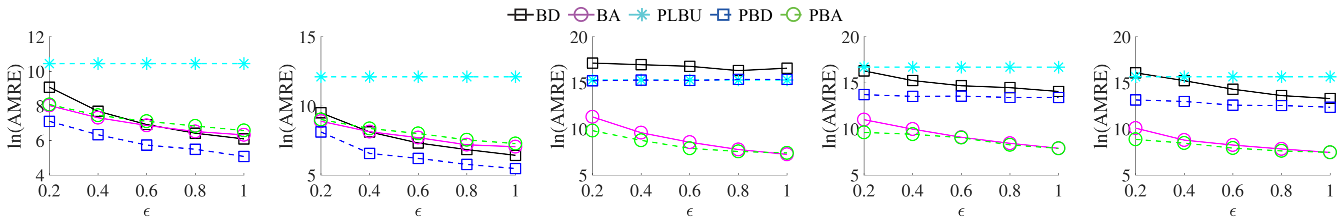
Figure 10 shows the natural logarithm of *AMRE* as the privacy budget ϵ varies. Across all datasets, *AMRE* decreases as ϵ increases, because a larger ϵ results in smaller noise variance, leading to a lower *AMRE*. The decrease in *AMRE* is more pronounced on real datasets compared to synthetic ones. It is because data density function changes rapidly in real datasets, while changing gradually in synthetic datasets. When the density function changes rapidly, the dissimilarity at each time slot becomes large. In this case, PBD publishes more new statistical results than PBA because PBD always reserves part of its privacy budget for the next time slot, even though the budget decreases over time within a window. Thus, PBD leads to higher accuracy than PBA. When the density function changes gradually, the dissimilarity at each time slot remains small. In this case, publishing one highly accurate statistical result at a time slot is more important than publishing multiple new statistical results. Therefore, PBA performs significantly better than PBD. PLBU performs worse than other methods across all datasets except for TLNS, since LDP methods achieve lower accuracy than CDP methods under the same privacy budget. In real datasets, our PBD consistently outperforms other methods. The *AMRE* of PBD is on average 70.8% (17.5% in terms of ln (*AMRE*)) lower than that of BD on Taxi dataset and 69.6% (15.9% in terms of ln (*AMRE*)) lower on Foursquare dataset. Our PBA performs slightly worse than BA, since our PBA is more sensitive to noise in high-dimensional data. For synthetic datasets, our PBA consistently outperforms other methods. Compared to BA, the *AMRE* of PBA is lower on average of 36.9% (6.0% in terms of ln (*AMRE*)) on TLNS dataset, 27.7% (4.2% in terms of ln (*AMRE*)) on Sin dataset, and 28.9% (4.5% in terms of ln (*AMRE*)) on Log dataset. Moreover, our PBD consistently outperforms BD.

Figure 11 shows the natural logarithm of *AMRE* as the window size w varies. As w increases, *AMRE* rises gently, particularly on the synthetic datasets. This occurs because a large window size results in a small privacy budget at each time slot, leading to increased error. PLBU shows lower performance than other methods on all datasets except for TLNS, since LDP methods achieve lower accuracy than CDP methods under equivalent privacy budgets. For real datasets, our PBD achieves the lowest error compared to others methods. The *AMRE* of PBD is on average 63.1% (15.6% in terms of ln (*AMRE*)) lower than that of BD on Taxi dataset and 68.4% (16.5% in terms of ln (*AMRE*)) on Foursquare dataset. For synthetic datasets, our PBA demonstrates the lowest error among all methods. Compared to BA, the *AMRE* of PBA is lower by average of 35.1% (5.4% in terms of ln (*AMRE*)) for TLNS, 4.2% (0.4% in terms of ln (*AMRE*)) for Sin, and 16.6% (2.2% in terms of ln (*AMRE*)) for Log. Moreover, our PBD consistently outperforms BD across all datasets.

In summary, our PBD demonstrates superior performance on real datasets, with 68% smaller *AMRE* on average than BD. For synthetic datasets, our PBA outperforms BA with 24.9% smaller *AMRE* on average.

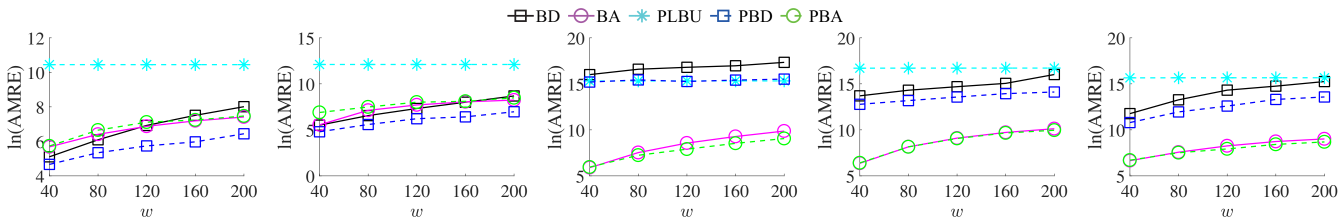
5.5 Impact of User Requirement Type

We define a set of users with privacy requirement as (w_k, ϵ_k)-*requirement type*. In this subsection, we examine the impact of user type on the utility.



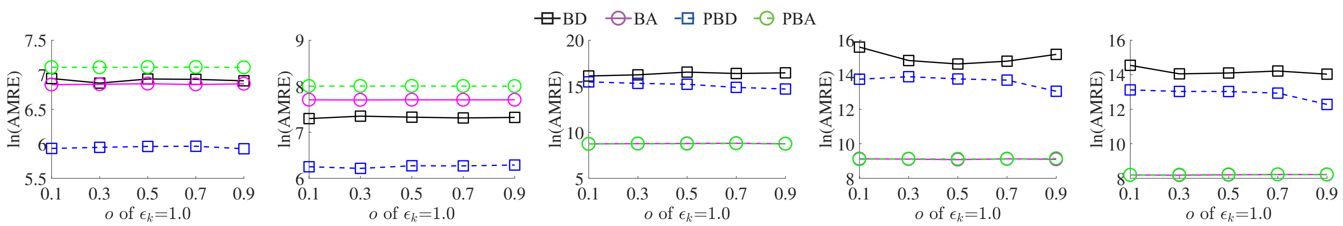
(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 10: *AMRE* with ϵ varied.



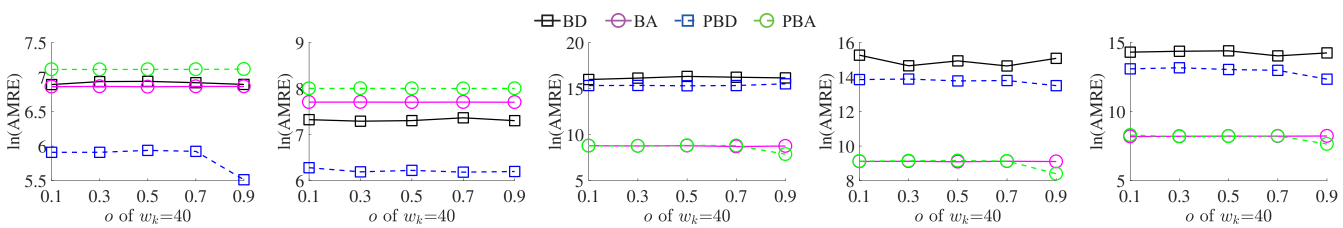
(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 11: *AMRE* with w varied.



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 12: *AMRE* with ratio for privacy budget varied.



(a) Taxi (b) Foursquare (c) TLNS (d) Sin (e) Log

Figure 13: *AMRE* with ratio for window size varied.

For the analysis, we consider $\epsilon_k \in \{0.6, 1.0\}$ with a default of 0.6, and the $w_k \in \{40, 120\}$ with a default of 120. We first vary the users' quantity ratio of $\epsilon_k = 1.0$ from 0.1 to 0.9 while keeping $w_k = 120$. Then we vary the users' quantity ratio of $w_k = 40$ from 0.1 to 0.9 while keeping $\epsilon_k = 0.6$.

Figure 12 illustrates the change in users' quantity ratio for $\epsilon_k = 1.0$ from 0.1 to 0.9, with a fixed window size of $w_k = 120$. Figure 13 shows the effect on changing users' quantity for $w_k = 40$ from 0.1 to 0.9, with a fixed privacy budget of $\epsilon_k = 0.6$. We observe that as the users' quantity ratio increases, the *AMRE* remains relatively stable. However, when the users' quantity ratio of $\epsilon_k = 1.0$ or $w_k = 40$ exceeds 0.8, we can see a significant decrease in *AMRE* for PBD and PBA. This occurs because when the ratios surpasses a certain threshold, the optimal budget from OBS in Algorithm 1 becomes dominated by a higher ϵ, resulting in lower error.

6 CONCLUSION

We address the Personalized w-Event Private Publishing for Infinite Data Streams problem by proposing a mechanism called Personalized Window Size Mechanism (PWSM). Based on PWSM, we develop two methods: Personalized Budget Distribution (PBD) and Personalized Budget Absorption (PBA). We evaluate both methods against recent solutions, Budget Distribution (BD) and Budget Absorption (BA), to demonstrate their efficiency and effectiveness. Our results show that PBD reduces error by 68% compared to BD on real datasets, while PBA achieves 24.9% lower error than BA on synthetic datasets.

7 ACKNOWLEDGMENT

Peng Cheng's work is partially supported by NSFC under Grant No. 62102149 and the Fundamental Research Funds for the Central Universities. Lei Chen's work is partially supported by National Key Research and Development Program of China Grant No. 2023YFF0725100, National Science Foundation of China (NSFC) under Grant No. U22B2060, Guangdong-Hong Kong Technology Innovation Joint Funding Scheme Project No. 2024A0505040012, the Hong Kong RGC GRF Project 16213620, RIF Project R6020-19, AOE Project AoE/E-603/18, Theme-based project TRS T41-603/20R, CRF Project C2004-21G, Guangdong Province Science and Technology Plan Project 2023A0505030011, Guangzhou municipality big data intelligence key lab, 2023A03J0012, Hong Kong ITC ITF grants MHX/078/21 and PRP/004/22FX, Zhujiang scholar program 2021JC02X170, Microsoft Research Asia Collaborative Research Grant, HKUST-Webank joint research lab and 2023 HKUST Shenzhen-Hong Kong Collaborative Innovation Institute Green Sustainability Special Fund, from Shui On Xintiandi and the InnoSpace GBA. Heng Tao Shen is supported by the Fundamental Research Funds for the Central Universities. Xuemin Lin is supported by NSFC U2241211. Corresponding author: Peng Cheng.

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