

# Consistent Range Approximation for Fair Predictive Modeling

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#### **ABSTRACT**

This paper proposes a novel framework for certifying the fairness of predictive models trained on biased data. It draws from query answering for incomplete and inconsistent databases to formulate the problem of consistent range approximation (CRA) of fairness queries for a predictive model on a target population. The framework employs background knowledge of the data collection process and biased data, working with or without limited statistics about the target population, to compute a range of answers for fairness queries. Using CRA, the framework builds predictive models that are certifiably fair on the target population, regardless of the availability of external data during training. The framework's efficacy is demonstrated through evaluations on real data, showing substantial improvement over existing state-of-the-art methods.

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The source code, data, and/or other artifacts have been made available at https://github.com/lodino/Crab.

### 1 INTRODUCTION

Algorithmic decision-making systems are increasingly prevalent in critical domains, highlighting the importance of fairness. The objective is to ensure equal treatment across diverse groups based on sensitive attributes. Consequently, research in machine learning (ML), data management, and related fields has grown to address algorithmic fairness [4, 35]. This paper particularly focuses on predictive modeling, aiming to train ML models that provide accurate predictions while ensuring fairness. Various metrics, such equality of odds, are used to evaluate fairness [18, 33, 58].

Traditional fairness methods in predictive modeling can be categorized as in-processing or pre-/post-processing techniques [15, 26, 27, 52, 62, 76, 83]. However, these methods often assume that the training data is representative of the target population [35]. In practice, **biases in data and quality issues** during data collection and preparation can distort the underlying data distribution, rendering it no longer representative of the target population. As a

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result, deploying these models in the target population may lead to unfair and inaccurate predictions [6, 31, 35, 37, 48].

A significant issue in predictive models is **selection bias**, resulting from training data selection based on specific attributes, which creates unrepresentative datasets. This problem is prevalent in sensitive areas like predictive policing, healthcare, and finance, attributed to data collection costs, historical discrimination, and biases [13, 20, 32, 40]. For example, in predictive policing, the data is biased as it is gathered exclusively from police interactions, which are influenced by the sociocultural traits of the officers [28, 43]. Similarly, in healthcare, selection bias occurs when data is relied upon from individuals who are hospitalized or have tested positive, leading to disproportionate effects on racial, ethnic, and gender minorities due to barriers in healthcare access [2, 16, 65, 88].

Example 1.1. Consider the dataset in Table 1, which represents a sample collected from a population of individuals. The objective is to train a ML model to predict future crime risk based on various features while ensuring fairness according to the equality of odds principle. This principle aims to maintain similar true positive and false positive rates across different protected and sensitive groups. However, our access to data is limited to that collected by police departments, resulting in a biased dataset. The dataset only includes records selected by law enforcement, indicated by the variable C, while records where C=0 are not available (highlighted in red).

Using incomplete data for training an ML model has two significant ramifications. First, the selected subset of data may exhibit a spurious correlation between certain attributes (e.g., race, gender, age) and the label, which either does not exist or is not as significant in the complete dataset. This means that a classifier trained on this incomplete dataset learns this misleading correlation, resulting in subpar performance when applied to the entire dataset in terms of accuracy and fairness. For example, in the given case, although age and the training labels are not correlated in the complete data, they become correlated in the selected subset of data.

Second, solely relying on existing fair ML techniques to train a model that is fair on a selected subset of data does not guarantee fairness on the complete data or the target population where the model will be deployed. In the provided example, although the model's predictions on the selected subset demonstrate accuracy and fairness according to the equality of odds principle with respect to race, its performance deteriorates when applied to the complete dataset, resulting in inaccurate and unfair predictions (see Example 3.1 for more details).

Recent efforts to mitigate selection bias in ML often rely on accessing unbiased samples from the target population (e.g., [5, 19, 34, 44, 60, 71]). However, obtaining unbiased samples from sensitive domains like predictive policing, healthcare, and finance poses significant challenges due to inherent biases specific to each domain.

**Table 1:** A toy dataset demonstrating selection bias in Example 1.1. Circled cells correspond to wrong predictions.

Crime Type	Age	Race	ZIPCode	С	Y (label)	Prediction
Arson	Young	Black	90043	1	High Risk	High Risk
Homicide	Young	Black	90043	1	High Risk	High Risk
Theft	Old	Black	90043	1	Low Risk	Low Risk
Robbery	Young	Black	90043	1	High Risk	High Risk
Car Break-in	Old	White	90026	0	High Risk	Low Risk
Assault	Old	White	90026	1	Low Risk	Low Risk
Theft	Young	White	90026	0	Low Risk	High Risk
Assault	Young	Black	90026	0	Low Risk	High Risk
Armed Robbery	Old	White	90026	1	High Risk	High Risk

Furthermore, methods that rely on biased samples have demonstrated poor real-world performance [19, 44], often neglecting fairness concerns in predictive modeling.

The common theme in data collection across sensitive domains like predictive policing, healthcare, and finance is that while obtaining unbiased data may be infeasible, it's feasible to acquire background knowledge about the data collection process. In predictive policing, data factors include demographic characteristics, sociocultural traits, and residence. In healthcare, datasets reflect age, race, socio-economic status, and the selection process. For finance, credit risk or loan approval datasets are influenced by access to financial services, income, employment status, and location. Additionally, it's possible to gather partial information about the target population from external data sources such as census data, open knowledge graphs, and data lakes. In healthcare, external sources like government databases provide unbiased aggregated demographic, socioeconomic, and geographic information, including healthcare resources access and insurance coverage. For predictive policing and finance, external sources like crime statistics, open data lakes, and credit bureau data offer unbiased insights into race, socio-economic status, and credit history. In finance, credit bureau data, financial institution databases, and government statistics provide valuable unbiased demographic, income, employment status, and location information.

In this work, we propose a novel framework called Consistent Range Approximation from Biased Data (CRAB) to address the challenge of constructing certifiably fair predictive models in the presence of selection bias, even when obtaining unbiased samples from the target population is not feasible. The key idea of CRAB is to leverage background knowledge on the data collection process, encoded through a causal diagram representing the dependence between the selection of data points and their corresponding features. Moreover, Crab can incorporate external data sources that provide additional information about the target population to enhance its results. By understanding the data collection process, CRAB formulates conditions that enable the training of predictive models that are certifiably fair on the target population. Unlike previous techniques that rely on unbiased samples and do not explicitly address fairness, CRAB ensures fairness even in the absence of unbiased data during model training and testing.

Example 1.2. Continuing with example 1.1, our system Crab takes as input the incomplete data (the selected subset) in Table 1, along with background knowledge that indicates the selection of data points is dependent on individual Zipcodes and other sociocultural traits. This information is encapsulated using a simple

causal model that encodes that the variable C is a function of factors such as Zipcode and other socio-cultural traits (This will be further elucidated in Section 2.1, Figure 1). Our system can also incorporate potential external data sources that contain unbiased information about the distribution of sensitive attributes in Zipcodes and so forth. Subsequently, the system trains an ML model using the incomplete data, ensuring that it will be fair on the complete data and, consequently, the target population.

Selection bias presents a fundamental challenge in training fair predictive models, as it prevents the accurate measurement of **fairness queries** - aggregate queries used to quantify fairness violations in an ML model on the target population, such as equality of odds. For instance, in the case of equality of opportunity, the fairness query is an aggregate query in the range [0,1] that assesses the disparity in the likelihood of a positive outcome between privileged and protected groups. This problem arises because **selection bias results in incomplete and inconsistent data**, making fair ML modeling essentially a **data management problem** concerning query answering from incomplete data. While query answering from inconsistent and incomplete data has been extensively studied in the database field [12, 22, 25], it hasn't been specifically addressed for biased data, apart from [55], which assumes access to unbiased data samples.

To tackle this issue, we draw insights from data management to introduce and formalize the problem of Consistent Range Approximation (CRA) of fairness queries from biased data. This approach aims to approximate the fairness of an ML model on a target population using background knowledge about the data collection process and limited or no information from external data sources. Inspired by Consistent Query Answering in databases [12, 22], CRA considers the space of all possible repairs that are consistent with the available information. It uses this to compute a range for fairness queries such that the true answers are guaranteed to lie within the range. We refer to this as the consistent range. (Section 3.1)

We present a closed-form solution for the problem of Consistent Range Answer (CRA) fairness queries in predictive modeling. Our analysis focuses on a class of aggregate queries that capture different notions of algorithmic fairness, such as statistical parity, equality of odds, and conditional statistical parity. We demonstrate that the consistent range can be efficiently calculated by incorporating varying levels of information about the target population from external data sources. This approach allows us to estimate the fairness of a model on the target population using biased data. Our results facilitate both the verification of approximate fairness and the training of certifiably fair models on all populations consistent with the available information about the target population, including the target population itself. The ability of CRA to accommodate varying levels of external data sources makes it a practical solution for addressing selection bias (Section 3).

Furthermore, we conduct a theoretical analysis of the impact of selection bias on the fairness of predictive models and establish necessary and sufficient conditions on the data collection process under which selection bias leads to unfair predictive models. Our results indicate that selection bias does not necessarily lead to unfair models, and in situations where it does, existing techniques are often inapplicable for training fair classifiers. This highlights the

importance of addressing selection bias in the data management stage, rather than relying on post-processing methods (Section 4).

We evaluate CRAB on both synthetic and real data. Our findings show that when selection bias is present: (1) existing methods for training predictive models result in unfair models. (2) In contrast, CRAB develops predictive models that are guaranteed to be fair on the target population. (3) Even when limited external data about the target population is available, CRAB still produces fair and highly accurate predictive models. (4) In certain situations, enforcing fairness can also improve the performance of predictive models. (5) Interestingly, the predictive models developed by CRAB in the presence of limited external data outperform those trained using current methods that have access to complete information about the target population (Section 5).

This paper is organized as follows: In Section 2, we provide background information on fairness, causality, and selection bias. In Section 3.1, we introduce and study the problem of CRA fairness queries, and we establish conditions under which it is possible to train certifiably fair ML models with varying access to external data sources about the target population. In Section 4, we establish sufficient and necessary conditions under which fairness leads to unfair predictive models. Finally, in Section 5, we provide experimental evidence that CRAB outperforms the state-of-the-art methods for training fair ML models and learning from biased data. An extended version of this paper including the missing proofs and additional experiments can be found in the extended version [89].

#### 2 PRELIMINARIES AND BACKGROUND

We now review the background on ML, causality, and selection bias. Table 2 shows the notation we use.

**Table 2:** Notation used in this paper.

Symbol	Meaning					
X, Y, Z X, Y, Z x, y, z	Attributes (features, variables) Sets of attributes Attribute values					
$ \begin{array}{c c} Dom(X) \\ G \\ D^{tr}, D^{ts} \\ D_{O} \end{array} $	Domain of an attribute A causal diagram Training/testing datasets Data sampled from distribution Ω					
$ \begin{array}{c c}  & MB(X) \\  & h(x) \end{array} $	Markov Boundary of a variable X A classifier					

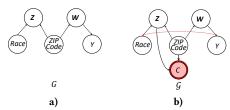
In this work, we focus on the problem of binary classification, which has been the primary focus of the literature on algorithmic fairness. Consider a population or data distribution  $\Omega$  with support  $X \times Y$ , where X denotes a set of discrete and continuous features and  $Dom(Y) = \{0, 1\}$  represents some binary outcome of interest (aka the target attribute). A classifier  $h: Dom(X) \rightarrow \{0,1\}$ is a function that predicts the unknown label y as a function of observable features x. The quality of a classifier h can be measured using the *expected loss*, also known as the *risk*, i.e., Loss(h) = $\mathbb{E}_{\Omega}[L(h(x),y)]$ , where L(h(x),y) is a loss function that measures the cost of predicting h(x) when the true value is y. In this paper, we focus on the zero-one loss, i.e.,  $L(h(x), y) = \mathbb{1}(h(x)) \neq \mathbb{1}(h(x))$ y). A learning algorithm aims to find a classifier  $h^* \in \mathcal{H}$  that has a minimum loss, i.e., for all classifiers  $h \in \mathcal{H}$ , it holds that  $Loss(h^*) \leq Loss(h)$ , where  $\mathcal{H}$  denotes the hypothesis space. In the case of the zero-one loss, the optimal classifier  $h^*$  is called the Bayes optimal classifier and is given by  $h^*(x)$  iff  $\Pr(y=1\mid x)\geq \frac{1}{2}$ . Since  $\Omega$  is unknown, we cannot calculate Loss(.) directly. Instead, given an i.i.d. sample  $D_{\Omega}^{tr} = \{(x_i, y_i)\}_{i=1}^n$  from  $\Omega$ , the *empirical loss*  $\frac{1}{n} \sum_{i=1}^n L(h(x), y)$  is typically used to estimate the expected loss. However, minimizing the empirical loss for the zero-one loss function is NP-hard due to its non-convexity. As a result, a convex surrogate loss function is used by learning algorithms. A surrogate loss function is said to be Bayes-risk consistent if its corresponding empirical minimizer converges to the Bayes optimal classifier when training data is sufficiently large [7].

Algorithmic fairness. Algorithmic fairness, particularly in the context of a binary classifier h with a protected attribute  $S \in X$  (e.g., gender or race), aims to ensure non-discriminatory predictions. In this setting, we define h(.) = 1 as a favorable prediction and h(.) = 0 as an unfavorable one. For simplicity, we denote  $\Pr(h(.) = 1)$  as  $\Pr^+(h(.))$ . We assume that  $\operatorname{Dom}(S) = \{s_0, s_1\}$ , representing a privileged group  $(s_1)$  and a protected group  $(s_0)$ . We focus on  $\operatorname{Conditional Statistical Parity}[78]$ , which requires equal positive classification probabilities for both groups, conditioned on admissible features A. These admissible features, such as a person's past convictions in predictive policing, are non-discriminatory and thus permissible for decision-making. Formally, this definition necessitates that for all admissible feature values  $a \in \operatorname{Dom}(A)$ , we have  $\Pr^+_{\Omega}(h(x) \mid s_0, A = a) = \Pr^+_{\Omega}(h(x) \mid s_1, A = a)$ .

Conditional statistical parity is a versatile fairness notion that captures many other fairness principles. When the set of admissible features is empty ( $A = \emptyset$ ), it reduces to *Statistical Parity* [24], which requires equal positive classification probabilities for the protected and privileged groups. Conversely, when the set of admissible features includes all available features (A = Y), it encompasses Equality of Opportunity [33], which demands equal positive prediction probabilities for both groups among individuals who should have received a favorable outcome based on the ground truth. Therefore, satisfying conditional statistical parity addresses both statistical parity and equality of opportunity. Furthermore, the methods developed in this paper can be extended and adapted to handle other fairness notions, such as causal notions [61].

#### 2.1 Background on Selection Bias

Selection bias occurs when the selection of a data point in a sample from a data distribution is not based on randomization, but rather on certain attributes of the data point. This results in a sample that is not representative of the data distribution. To address this issue, we use causal diagrams to encode background knowledge about the data collection process and its potential biases. This approach allows us to model and analyze selection bias in a principled way. Next, we give a short overview of how we employ causal diagrams. 2.1.1 Causal diagrams. A causal diagram is a directed graph that represents the causal relationships between a collection of observed or unobserved (latent) variables X and models the underlying process that generated the observed data. Each node in a causal diagram corresponds to a variable  $X \in X$ , and an edge between two nodes indicates a potential causal relationship between the two variables. The set of all parents of a variable X is denoted by Pa(X). d-separation and collider bias. Causal diagrams encode a set of conditional independences that can be read off the graph using dseparation [57]. A path is a sequence of adjacent arcs, e.g., (Race  $\rightarrow$  $Z \leftarrow ZIPCode$ ) in Figure 1a. Two nodes are d-separated by a set of



**Figure 1:** (a) A causal diagram G for predictive policing in Examples 2.1. (b) Data collection diagram G of police data in Example 2.2 in the presence of selection bias.

variables  $V_m$  in causal diagram G, denoted  $(V_l \perp V_r \mid_d V_m)$  if for every path between them, one of the following conditions holds: (1) the path contains a chain  $(V_l \rightarrow V \rightarrow V_r)$  or a fork  $(V_l \leftarrow V \rightarrow V_r)$  such that  $V \in V_m$ , and (2) the path contains a collider  $(V_l \rightarrow V \leftarrow V_r)$  such that  $V \notin V_m$ , and no descendants of V are on  $V_m$ . For example, Y and X are  $X_m$  are  $X_m$  and  $X_m$  and  $X_m$  in  $X_m$  in  $X_m$  in  $X_m$  and  $X_m$  in  $X_$ 

**Conditional Independence.** A data distribution  $\Omega$  is said to be *Markov compatible*, or simply compatible, with a causal diagram G if d-separation over the G implies conditional independence with respect to  $\Omega$ . More formally,  $(X \perp\!\!\!\perp Y \mid_d Z) \implies (X \perp\!\!\!\perp_\Omega Y \mid Z)$ , where  $(X \perp\!\!\!\perp_\Omega Y \mid Z)$  means X is independent of Y conditioned on Z in  $\Omega$ . If the converse also holds, i.e.,  $(X \perp\!\!\!\perp_\Omega Y \mid Z) \implies (X \perp\!\!\!\perp Y \mid_d Z)$ ,  $\Omega$  is considered *faithful* to G.

Example 2.1. Figure 1a shows a simplified causal model for Example 1.1) with variables Y: drug use; W: variables such as income, education, job that are deemed to causally affect drug use; Z: sociocultural traits, zip code, and race. For a distribution  $\Omega$ compatible with this causal diagram, it holds that  $(Race \perp \!\!\! \perp_{O} Y)$  because Race and Y are d-separated by an empty set since the path  $(Race \rightarrow Z \leftarrow ZIPCode \rightarrow W \rightarrow Y)$  is closed at a collider node Z. 2.1.2 Data collection Diagrams. We use causal diagrams to represent selection bias in data collection. Given a target population  $\Omega$  that is faithful to a causal diagram G, we can model a biased data collection process (where the selection of data points depends on a set of variables  $V \subseteq X \cup \{Y\}$ ) using a data collection diagram G. This is achieved by augmenting G with a selection node C, where V constitutes the parents of C, i.e. V = Pa(C). In this way, the collected data  $D_{\Delta}$  can be seen as an i.i.d sample from a biased data distribution  $\Delta$  that is compatible with  $\mathcal{G}$ , such that  $\Pr_{\Lambda}(x, y) = \Pr_{\Omega}(x, y \mid C = 1)$ . Additionally, conditioning on C in the biased data distribution may result in a spurious correlation between variables for which C is a collider in a path between them. We provide an example to illustrate this concept.

Example 2.2. (Example 2.1 continued) Figure 1b shows a data collection diagram for a scenario in which the selection of data points into a sample depends on the neighborhood that is more regularly patrolled and individuals' sociocultural traits, as indicated by arrows from the Z and ZIPCode variables to the selection variable C. A sample of data collected according to this biased data collection process can be seen as a sample from a biased data distribution  $\Pr_{\Delta}(X) = \Pr_{\Omega}(X \mid C = 1)$ , where  $X = \{W, Z, Y, Race, ZIPCode\}$ . Furthermore, since C is a collider in the path ( $Race \rightarrow Z \rightarrow C \leftarrow ZIPCode \rightarrow W \rightarrow Y$ ) between Race and Y, conditioning on the selection variable C induces a spurious

correlation between Race and Y that does not exist in the target distribution  $\Omega$ , but exists in the biased data  $\Delta$ , due to the collider bias, as indicated by the red bidirectional arrow between Race and Y in the diagram. Specifically, while in the target data distribution  $\Omega$  it holds that  $(Race \coprod_{\Omega} Y)$ , in the biased data distribution  $\Delta$   $(Race \coprod_{\Delta} Y)$ . Hence, a sample  $D_{\Delta}$  from  $\Delta$  may exhibit spurious correlations between Race and Y that may lead unfair predictive models trained on  $D_{\Delta}$ . Additionally, the data collection diagram encodes the independence of race, W, and Y from the selection variable given Z and ZIP code. This means that the conditional distribution  $\Pr_{\Omega}(Race, W, Y \mid Z, ZIPCode)$  of the target population can be calculated from biased data, and therefore biased data can provide some information about the target population's statistics. This is because  $\Pr_{\Omega}(Race, W, Y \mid Z, ZIPCode) = \Pr_{\Omega}(Race, W, Y \mid Z, ZIPCode)$ .

# 3 FAIR CLASSIFICATION UNDER SELECTION BIAS USING CRAB

The presence of selection bias can result in a discrepancy between the target distribution  $\Omega$  and the biased data distribution  $\Delta$ . This variance in the two distributions can result in a difference in the unfairness of a classifier evaluated on each. For instance, a classifier that is fair on the biased training data can still exhibit unfairness when deployed on the actual data distribution [48].

Example 3.1. Consider the classifier, h(Age, Crime Type) = High Risk if (Crime Type = Armed Robbery) or (Age = Young), otherwise Low Risk, as presented in Example 1.1 using Table 1. This classifier, derived from biased data, seems fair in terms of equality of odds and appears accurate when applied to the selected subset used for training. Although it's feasible to devise a perfect and fair classifier based solely on crime type, this model capitalizes on the observed correlation between age and high risk within the selected subset, creating a seemingly flawless and fair model. However, when this classifier is applied to the entire dataset, it manifests as unfair, despite its predictions not being directly based on race.

Our goal is to develop ML models that can effectively handle selection bias while ensuring fairness during deployment on the target distribution. We formally define this objective as follows:

Definition 3.1 (Fair classifier). A classifier h trained on a training data  $D_{\Delta}^{tr}$  sampled under selection bias from a biased distribution  $\Delta$  to predict the class label Y is fair if it satisfies conditional statistical parity on an (unseen) test data  $D_{\Omega}^{ts}$  that is a representative sample of the target distribution  $\Omega$ .

It is widely recognized in the literature that learning a fair classifier without any auxiliary information about the target distribution or the data collection process is practically impossible [17, 21, 29, 67, 77]. Without any auxiliary information to serve as constraints, the target distribution may deviate significantly from the distribution observed in the training data. This is illustrated in Example 3.1. The importance of having knowledge about the target distribution has been acknowledged in the literature [51, 63, 70].

To address this challenge, first we introduce the problem of consistent range approximation (CRA), which aims to approximate and bound the answers to fairness queries on a target population using biased data (Section 3.1). Then, we study the problem

of CRA in settings for which limited or no external data source about the target population is available (Section 3.1.1-3.1.2). The CRA framework for fairness queries can be combined with standard approaches, such as incorporating fairness constraints or regularization into the learning process. By adding upper bounds of fairness queries as constraints or regularization terms, the model is penalized to ensure it has an approximately zero consistent upper bound. This ensures fairness across all possible repairs and ultimately extends to the target population (Section 3.2).

# 3.1 Consistent Range Approximation (CRA) of a Fairness Query

In this work, we utilize the total variation distance as a measure for evaluating the degree of fairness violation of a classifier h, wrt. the fairness definitions outlined in Section 2. This approach encompasses a variety of other methods found in the literature for identifying discrimination (see, e.g., [3, 10, 42, 45, 49, 60, 69, 82–86]). Specifically, we formulate the fairness query as follows:

Definition 3.2 (Fairness Query). We measure the fairness violation of a classifier h with respect to a given set of admissible attributes A and on a population  $\Omega$  with support  $X \times Y$  using a fairness query defined as follows:

$$\int_{h,A}(\Omega) = \frac{1}{2|A|} \sum_{\substack{y \in DOM(Y), \\ \boldsymbol{a} \in DOM(A)}} |\operatorname{Pr}_{\Omega}(h(\boldsymbol{x}) = y \mid s_1, \boldsymbol{a}) - \operatorname{Pr}_{\Omega}(h(\boldsymbol{x}) = y \mid s_0, \boldsymbol{a})|.$$

The fairness query in Eq (1) quantifies the level of unfairness of a classifier h by comparing the average total variation distance between the conditional probability distributions  $\Pr_{\Omega}^+(h(x)\mid s_1,a)$  and  $\Pr_{\Omega}^+(h(x)\mid s_0,a)$  for all  $a\in \mathrm{Dom}(A)$ . A classifier h is considered  $\epsilon$ -fair on population  $\Omega$  if  $f_{h,A}(\Omega)\leq \epsilon$ . Notably, Eq (1) measures discrimination for a  $x\in \mathrm{Dom}(X)$ . The overall unfairness can be calculated by taking the expectation over x. It's important to note that a classifier that satisfies conditional statistical parity on  $\Omega$  would have a value of  $f_{h,A}(\Omega)=0$ . For brevity, in subsequent sections we will refer to the fairness query by omitting explicit references to  $\Omega$ , h, and A when the context is clear, e.g., using the symbol f.

Our focus is on binary classification and, for simplicity, we assume  $\Pr^{\perp}_{\Omega}(h(x) \mid s_1, a) \geq \Pr^{\perp}_{\Omega}(h(x) \mid s_0, a)$  holds for arbitrary  $a \in A$  throughout this section. However, this assumption is not restrictive and can be easily adapted during implementation without compromising the generality of our results. Under this assumption, the fairness query in Eq (1) simplifies as follows:

$$f(\Omega) = \frac{1}{|A|} \sum_{\boldsymbol{a} \in \text{Dom}(\boldsymbol{A})} \text{Pr}_{\Omega}^{+}(\boldsymbol{h}(\boldsymbol{x}) \mid s_{1}, \boldsymbol{a}) - \text{Pr}_{\Omega}^{+}(\boldsymbol{h}(\boldsymbol{x}) \mid s_{0}, \boldsymbol{a})$$
(2)

In practice, the fairness query in Eq (1) must be computed using data through an *empirical fairness query* denoted  $\hat{f}$ , which is typically defined based on the empirical estimate as follows:

Definition 3.3 (Empirical Fairness Query). Fairness violation of a classifier h on a dataset  $D_{\Omega}$  sampled from a distribution  $\Omega$  can be obtained by the following empirical fairness query:

$$\hat{\mathbf{f}}(D_{\Omega}) = \frac{1}{|A|} \sum_{\boldsymbol{a} \in DOM(A)} \left| \frac{\sum_{\boldsymbol{x} \in N_{s_{1},\boldsymbol{a}}^{+}} h(\boldsymbol{x})}{|N_{s_{1},\boldsymbol{a}}^{+}|} - \frac{\sum_{\boldsymbol{x} \in N_{s_{0},\boldsymbol{a}}^{+}} h(\boldsymbol{x})}{|N_{s_{0},\boldsymbol{a}}^{+}|} \right|$$
(3)

where  $s_1$  and  $s_0$  are the protected attributes, and  $N_{s,a}^+$  denotes the set of data points in  $D_{\Omega}$  with positive labels, protected attribute value S = s and admissible attributes value A = a.

Note that  $\hat{f}(D_{\Omega})$  is an unbiased estimate of  $f(\Omega)$ , and is often used as it can be estimated from the data (see, e.g., [8, 38, 41, 83] and extended version [89] for details). In order to avoid sampling variability, in the subsequent, we assume samples are sufficiently large such that  $\hat{f}(D_{\Omega}) \approx f(\Omega)$  and use them interchangeably.

The problem lies in the fact that we do not have access to the true data,  $D_{\Omega}$ , and the only data we have is  $D_{\Delta}$ , which is drawn from a biased distribution  $\Delta$ . This means that the empirical fairness estimate,  $\hat{f}(D_{\Delta})$ , can provide a biased and incorrect estimate of the fairness of a classifier on the target population,  $f(\Omega)$ . This is a concern even if the sample  $D_{\Delta}$  is large, as  $f(\Delta)$  and  $f(\Omega)$  are generally not equal. Our aim is to approximate and bound  $f(\Omega)$  using the biased  $D_{\Delta}$  and auxiliary information regarding the data collection process and target population.

Definition 3.4 (Auxiliary Information). The auxiliary information  $I_{\Omega} = (\mathcal{G}, \mathcal{A}_{\Omega})$  is a tuple where  $\mathcal{G}$  is the data collection diagram that represents the underlying biased data collection process, and  $\mathcal{A}_{\Omega}$  is a set of external data sources that can potentially provide partial information about the target population  $\Omega$ .

We now introduce our problem setup.

Definition 3.5 (Fairness Query Application). A fairness query application is a tuple  $(h,f,D_{\Delta},I_{\Omega})$ , where h is a classifier trained on a dataset  $D_{\Delta}$  that was collected under selection bias and corresponds to a biased distribution  $\Delta$ , f is a fairness query, and  $I_{\Omega}$  is the auxiliary information about the data collection process and the target population  $\Omega$ .

In this work, we formulate the problem of information incompleteness caused by selection bias by using the concept of *possible repairs* from Consistent Query Answering [12, 22]. This setup allows us to account for the uncertainty in the information about the target population  $\Omega$  and bound the estimates for the unfairness of the classifier on  $\Omega$ . It is important to note that we do not actually compute the repairs themselves, but rather use the concept of possible repairs as a framework for addressing the incompleteness of information in the presence of selection bias.

DEFINITION 3.6 (POSSIBLE REPAIRS). Given a biased dataset  $D_{\Delta}$ , and auxiliary information  $I_{\Omega} = (\mathcal{G}, \mathcal{A}_{\Omega})$ , we define the set of possible repairs of  $D_{\Delta}$ , denoted as Repairs  $(D_{\Delta})$ , as the set of all datasets D with the same schema as  $D_{\Delta}$  such that: (1)  $D \supseteq D_{\Delta}$  and (2) D is consistent with  $\mathcal{G}$  and  $\mathcal{A}_{\Omega}$ , i.e., it satisfies the constraints specified by these two components of  $I_{\Omega}$ .

Intuitively, any repair in Repairs $(D_{\Delta})$  should be a superset of the biased dataset  $D_{\Delta}$ , adhering to the conditional independence constraints specified by  $\mathcal{G}$  and aligning with statistics derived from  $\mathcal{A}_{\Omega}$  pertaining to the target distribution  $\Omega$ . As we may have no or partial information about  $\Omega$  in  $\mathcal{A}_{\Omega}$ , there are potentially infinite ways to repair  $D_{\Delta}$ . These constraints ensure that the repaired data aligns with any known information about  $\Omega$ . Using this concept of possible repairs, we can now define the problem of consistently approximating a fairness query from biased data.

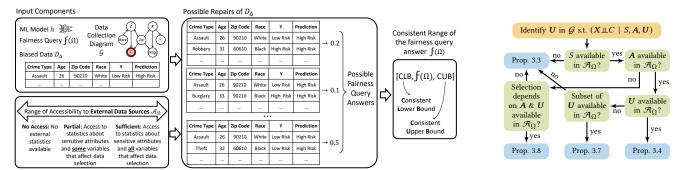


Figure 2: Left: An Illustration of the CRA process, which returns the range of possible answers  $\Omega$ . The range is determined by computing the greatest lower bound and the lowest upper bound of the answer to the empirical fairness query over the set of possible repairs, providing a measure of the classifier's unfairness on the target population. Right: Summary of scenarios where each propositions should be applied.

PROBLEM 3.1 (CONSISTENT RANGE APPROXIMATION). Given a fairness query application  $(h, \int, D_{\Delta}, I_{\Omega})$ , the problem of Consistent Range Approximation (CRA) of a fairness query  $\int \Omega$  is to determine the range of possible answers [CLB, CUB], where the CLB and CUB denote the consistent lower and upper bounds of the answer to the empirical fairness query  $\hat{f}$  over possible repairs in Repairs  $(D_{\Delta})$ , respectively. Specifically,

$$CLB = \min_{D \in \text{Repairs}(D_{\Delta})} \hat{f}(D), \ CUB = \max_{D \in \text{Repairs}(D_{\Delta})} \hat{f}(D)$$

The set of possible repairs Repairs  $(D_{\Delta})$  includes all datasets that are consistent with the auxiliary information  $I_{\Omega}$ , including a representative sample  $D_{\Omega}$  from the target distribution  $\Omega$ . This means that the answer to  $f(\Omega)$ , which measures the fairness violation of a classifier on the target distribution, is guaranteed to be included within the range [CLB, CUB] (See figure 2). This enables auditing the fairness of a classifier when the training data suffers from selection bias and limited information about the target distribution is available. One can also certify that a classifier is  $\epsilon$ -fair by checking that CUB  $\leq \epsilon$ . Additionally, as shown in Section 3.2, the upper bound can be used as a constraint or regularizer to train a classifier that is certifiably  $\epsilon$ -fair on the target population.

Next, we investigate methods for obtaining consistent range answers to fairness queries in scenarios where the external data source is either absent or only provides partial information about the target population. While it is not possible to fully characterize the set of all repairs, we establish conditions under which closedform solutions for consistent ranges can be obtained efficiently.

3.1.1 Absence of External Data Sources. When there are no external data sources available, meaning  $\mathcal{A}_{\Omega} = \emptyset$ , we prove a sufficient condition for being able to estimate a consistent range  $f(\Omega)$  using only the biased data  $D_{\Delta}$ . This range is represented by a closed-form formula, which makes it easy to calculate. In the context of algorithmic fairness, we're primarily interested in worst-case analysis, so we will focus on finding an upper bound for  $f(\Omega)$  using this closed-form formula. Before we move forward, we'll also show a simple sufficient condition for  $f(\Omega) = f(\Delta)$ , which means that the unfairness of a classifier on the target population can be obtained by computing the empirical fairness query on the biased data. Recall from the discussion below Definition 3.3 that we assume  $f(\Delta) = \hat{f}(D_{\Delta}^{tr})$ , but we are still unable to directly evaluate  $f(\Omega)$  with the biased training data  $D_{\Delta}^{tr}$ .

PROPOSITION 3.2. Given a fairness query application  $(h, f, D_{\Delta}, I_{\Omega})$ , if the conditional independence  $(X \perp\!\!\!\perp C \mid A, S)$  holds, then  $CLB = f(\Omega) = f(\Delta) = CUB$ .

The independence condition in Proposition 3.2 does not hold in realistic situations because it requires the selection variable C to only depend on the admissible attributes A and the protected attribute S, which is unrealistic. In most real-world scenarios, the selection variable is influenced by multiple factors, making it dependent on other variables as well. Therefore, in such situations, it's expected that  $f(\Omega) \neq f(\Delta)$ . Next, we will establish a condition under which  $f(\Omega)$  can be upper bounded using biased data.

Proposition 3.3. Given a fairness query application  $(h, f, D_{\Delta}, I_{\Omega})$ , the following holds for any set of variables  $U \subseteq X \cup \{Y\}$  such that  $(X \perp \!\!\! \perp C \mid S, A, U)$ :

$$0 \leq f(\Omega) \leq CUB$$

$$= \frac{1}{|A|} \sum_{\boldsymbol{a} \in DOM(A)} \left( \max_{\boldsymbol{u} \in DOM(U)} \operatorname{Pr}_{\Delta}^{+}(h(\boldsymbol{x}) \mid s_{1}, \boldsymbol{a}, \boldsymbol{u}) \right)$$

$$- \min_{\boldsymbol{u} \in DOM(U)} \operatorname{Pr}_{\Delta}^{+}(h(\boldsymbol{x}) \mid s_{0}, \boldsymbol{a}, \boldsymbol{u}).$$
(4)

PROOF SKETCH. The proposition can be proven by creating an upper bound for  $\Pr_{\Omega}^+(h(x) \mid s_1, a)$  and a lower bound for  $\Pr_{\Omega}^+(h(x) \mid s_0, a)$  in the definition of  $f_{h,A}(\Omega)$  in Eq (2). These bounds can be derived by employing the conditional independence assumption and the law of total probability. The comprehensive proofs of our results are available in the extended version [89].

The independence condition  $(X \perp C \mid S, A, U)$  in Proposition 3.3 can be satisfied by setting U = Pa(C), which forms the Markov Boundary of C and ensures independence. Hence, Even without external data, an upper bound on  $f(\Omega)$  can be computed using only the data collection diagram and biased training data.

3.1.2 Presence of External Data Sources. Now we establish conditions under which  $f(\Omega)$  can be estimated or tightly bounded from biased data when we have access to external data sources that reveal varying levels of information about the target population.

PROPOSITION 3.4. Given a fairness query application  $(h, f, D_{\Omega}, I_{\Omega})$ , if there exists a set of variables  $U \subseteq X \cup \{Y\}$  such that  $(X \perp \!\!\! \perp \!\!\! \perp C \mid S, A, U)$  holds, and where the auxiliary statistics  $\Pr_{\Omega}(u \mid s, a)$ , for all  $u \in Dom(U)$ ,  $a \in Dom(A)$ , and  $s \in \{s_0, s_1\}$  can be obtained using external data sources  $\mathcal{A}_{\Omega}$ , then the fairness

query  $f(\Omega)$  can be computed using the following equation:

$$CLB = \int(\Omega) = CUB$$

$$= \frac{1}{|A|} \sum_{\boldsymbol{a} \in Dom(A)} \sum_{\boldsymbol{u} \in Dom(U)} \left( \Pr_{\Delta}^{+}(h(\boldsymbol{x}) \mid s_{1}, \boldsymbol{a}, \boldsymbol{u}) \Pr_{\Omega}(\boldsymbol{u} \mid s_{1}, \boldsymbol{a}) \right)$$

$$- \Pr_{\Delta}^{+}(h(\boldsymbol{x}) \mid s_{0}, \boldsymbol{a}, \boldsymbol{u}) \Pr_{\Omega}(\boldsymbol{u} \mid s_{0}, \boldsymbol{a})$$
(5)

Eq (5) calculates  $\Pr_{\Delta}^+(h(x)\mid s,a,u)$ , which can be estimated from biased data. However,  $\mathcal{H}_{\Omega}$  must be used to compute  $\Pr_{\Omega}(u\mid s,a)$ . The independence assumption  $(X \perp \!\!\! \perp C \mid S,A,U)$  is crucial to apply Proposition 3.3 and 3.4. By setting U=Pa(C), i.e., the parents of the selection variable C in the data collection diagram G, we can always find variables that satisfy this assumption (cf. Section 3.1.1). Therefore, these propositions are applicable even when the entire data collection diagram G is unknown and we have access to information only about Pa(C).

However, when the data collection diagram  $\mathcal{G}$  is available, we can select a minimal set of variables U that satisfies the independence condition and for which auxiliary information in  $\mathcal{A}_{\Omega}$  is available. This facilitates applying the proposition in settings where  $\mathcal{A}_{\Omega}$  does not contain information about the entire set of variables in Pa(C). Specifically, whenever the selection variable depends on the training label Y, i.e.,  $Y \in Pa(C)$ , using the data collection diagram  $\mathcal{G}$  enables identifying a set of variables U that does not contain Y and still satisfy the independence condition. This enables the use of Proposition 3.4 even if no information about the label Y is available in  $\mathcal{A}_{\Omega}$ , which is often the case in practice. We will show next that such a set of variables always exists, hence one can establish an upper bound for a fairness notion in general, even if no external data source about the outcome variable Y is required.

PROPOSITION 3.5. Given a data collection diagram  $\mathcal{G}$ , there always exists a set of variables  $U \subseteq Pa(C) \cup MB(Y) \setminus \{C,Y\}$  that satisfy the conditional independence  $(X \perp \!\!\! \perp C \mid S, A, U)$ . (Note that Pa(C) represents the parents of C in  $\mathcal{G}$ , and MB(Y) denotes the Markov Boundary of Y in  $\mathcal{G}$ .)

Example 3.6. In the data collection diagram in Figure 1b, the conditional independence  $(X \perp C \mid S, A, U)$  holds for  $U = Pa(C) = \{Z, ZIPCode\}$ . This means that  $f(\Omega)$  can be estimated from a biased sample collected according to the diagram in Figure 1b and using auxiliary information to estimate statistics of the form  $\Pr_{\Omega}(u \mid s, a)$ . Now consider a similar scenario, except that the selection variable is a function of Y and ZIPCode, i.e.,  $Pa(C) = \{Y, ZIPCode\}$ . In this case, the independence condition holds for  $U = Pa(C) = \{Y, ZIPCode\}$ . However, if information about the label Y is not available in external data sources, one can select  $U = \{ZIPCode, W\}$  that satisfy the independence condition  $(X \perp C \mid S, A, U)$  and use Proposition 3.4 to compute  $f(\Omega)$ .

Propositions 3.3 and 3.4 examine different ends of the spectrum in terms of the availability of external data. The former requires no external data, while the latter requires sufficient external data for exact computation of a fairness query. In practice, one may have access to a level of external data that falls in between these two extremes. In such cases, it is important to note that the selection variable, C, may depend on a high-dimensional set of variables, and thus the set of variables U for which the conditions of Proposition 3.3 hold could also consist of a high-dimensional set of

variables. This may make it infeasible to collect auxiliary information for computing all the statistics  $\Pr_{\Omega}(u \mid s, a)$  needed for exact computation. Thus, in this case, we investigate the middle of the spectrum, where some auxiliary information about the target population is available but not enough for applying Proposition 3.4. We show that this limited amount of auxiliary information can be used to compute a tighter upper bound for  $f(\Omega)$  than that established in Proposition 3.3. Specifically, we consider similar assumptions as in Proposition 3.4, but in situations where external data sources have only partial information about the statistics  $\Pr_{\Omega}(u \mid s_0, a)$ .

Proposition 3.7. Given a fairness query application  $(h, f, D_{\Delta}, I_{\Omega})$  and a subset of variables  $U \subseteq X \cup \{Y\}$  such that  $(X \perp \!\!\! \perp C \mid S, A, U)$  holds in  $\mathcal{G}$ , if  $\mathcal{A}_{\Omega}$  only permit computation of the auxiliary statistics  $\Pr_{\Omega}(u' \mid s, a)$  for all  $u' \in Dom(U')$  and  $s \in \{s_0, s_1\}$  for some subset  $U' \subseteq U$ , then the following upper bound can be computed for  $f(\Omega)$ :

$$0 \leq f(\Omega) \leq CUB = \frac{1}{|A|} \sum_{\boldsymbol{a} \in DoM(A)} \sum_{\boldsymbol{u'} \in \boldsymbol{U'}} \left( \Pr_{\Omega}(\boldsymbol{u'} \mid s_1, \boldsymbol{a}) \max_{\boldsymbol{u^*} \in DoM(\boldsymbol{U} \setminus \boldsymbol{U'})} (\Pr_{\Delta}^+(h(\boldsymbol{x}) \mid s_1, \boldsymbol{a}, \boldsymbol{u'}, \boldsymbol{u^*})) \right.$$

$$\left. - \Pr_{\Omega}(\boldsymbol{u'} \mid s_0, \boldsymbol{a}) \min_{\boldsymbol{u^*} \in DoM(\boldsymbol{U} \setminus \boldsymbol{U'})} (\Pr_{\Delta}^+(h(\boldsymbol{x}) \mid s_0, \boldsymbol{a}, \boldsymbol{u'}, \boldsymbol{u^*})) \right)$$

$$(6)$$

Now we consider a scenario where external data source about the entire set of admissible variables A is unavailable, which can be a challenge when dealing with fairness definitions based on error rate balance, such as equality of odds, where the training label Y is included in A. In these cases, Propositions 3.3 and 3.7 can only be applied in the presence of auxiliary information about Y, which is difficult to acquire in practice. However, we show that under certain assumptions about the data collection process, it is still possible to bound  $f(\Omega)$  from biased data even in the absence of auxiliary information about A.

PROPOSITION 3.8. Let  $(h, f, D_{\Delta}, I_{\Omega})$  be a fairness query application and  $\mathcal{G}$  the corresponding data collection diagram. If  $A \cap Pa(C) = \emptyset$  in  $\mathcal{G}$ , i.e., data selection does not directly depend on the admissible variables A, then for a set of variables  $U \subseteq X \cup \{Y\}$  that satisfies the conditional independence  $(X \perp \!\!\! \perp \!\!\! \perp C \mid S, A, U)$ , if the target population statistics  $\Pr_{\Omega}(s, u)$  for all  $u \in Dom(U)$  and  $s \in \{s_0, s_1\}$  can be computed from external data sources  $\mathcal{A}_{\Omega}$ , then  $f(\Omega)$  can be computed using the following formula:

$$CLB = \mathbf{f}(\Omega) = CUB$$

$$= \frac{1}{|A|} \sum_{\boldsymbol{u} \in DOM(\boldsymbol{U})} (\operatorname{Pr}_{\Delta}^{+}(\boldsymbol{h}(\boldsymbol{x}) \mid s_{1}, \boldsymbol{a}, \boldsymbol{u}) \ w(s_{1}, \boldsymbol{u}, \boldsymbol{a})$$

$$- \operatorname{Pr}_{\Delta}^{+}(\boldsymbol{h}(\boldsymbol{x}) \mid s_{0}, \boldsymbol{a}, \boldsymbol{u}) \ w(s_{0}, \boldsymbol{u}, \boldsymbol{a})).$$

$$where \ w(\boldsymbol{u}, s, \boldsymbol{a}) = \frac{\sum_{\boldsymbol{x} \in DOM(\boldsymbol{X})} \operatorname{Pr}_{\Delta}(\boldsymbol{a}, \boldsymbol{x} \mid s, \boldsymbol{u}) \operatorname{Pr}_{\Omega}(s, \boldsymbol{u})}{\sum_{\boldsymbol{u}^{*} \in DOM(\boldsymbol{U})} \sum_{\boldsymbol{x} \in DOM(\boldsymbol{X})} \operatorname{Pr}_{\Delta}(\boldsymbol{a}, \boldsymbol{x} \mid s, \boldsymbol{u}^{*}) \operatorname{Pr}_{\Omega}(s, \boldsymbol{u}^{*})}$$

3.1.3 Summary of the results for the CRA of fairness query. In this section, we've introduced methods to address the Consistent Range Answer (CRA) of fairness queries  $f(\Omega)$  under various scenarios, depending on the availability of an external data source  $(A_{\Omega})$  and the requirements of the data collection diagram  $(\mathcal{G})$  (refer to Figure 2). Without an external data source, we depend solely on the data collection diagram and use Proposition 3.3 to estimate the CUB of  $f(\Omega)$ . When external data sources are available, providing statistics about admissible variables, sensitive attributes, and a variable set U that meets the conditional independence constraint,

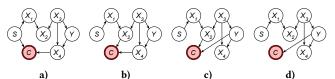


Figure 3: Examples that demonstrate our structural conditions.

we use Proposition 3.4 to directly approximate  $f(\Omega)$  (CUB=CLB). If only a subset of U is available from the external data source, we apply Proposition 3.7 to derive a tighter CUB using the available statistics. If the selection process doesn't depend on any admissible attribute, we only need external data sources about U and sensitive attributes to approximate  $f(\Omega)$  using Proposition 3.8.

#### 3.2 Fair ML with CRA

The results established for CRA of  $f(\Omega)$  from biased data can be used to train predictive models by solving the following constrained risk minimization problem (cf. Section 2), which enforces an upper bound on  $f(\Omega)$  obtained via CRA:

$$\min_{h \in \mathcal{H}} \mathbb{E}_{\Delta}[L(h(x), y)] \quad \text{s.t. CUB}(f(\Omega)) \le \tau$$
(8)

where  $\mathrm{CUB}(f(\Omega))$  is the consistent upper bound of a fairness query that can be computed from biased training data via the results established in Propositions 3.3 to 3.8, and  $\tau$  is a threshold that can be tuned to trade fairness with accuracy based on application scenarios. However, directly solving this constrained optimization problem for an arbitrary ML model is difficult. Instead, we can adopt the idea of the penalty method [8, 11, 39, 53, 68, 83] and convert the constrained optimization problem in Eq (8) to an unconstrained problem by adding a regularization term that penalizes high upper bound of unfairness to the objective function:

$$\min_{h \in \mathcal{H}} \mathbb{E}_{\Delta}[L(h(x), y)] + \lambda \cdot \text{CUB}(f(\Omega))$$
(9)

We use Algorithm 1 to solve the optimization problem in Eq (9). To balance fairness and accuracy, we introduce a fairness threshold  $\tau$  and use max  $(\text{CUB}(f(\Omega)), \tau)$  in the fairness constraint of the optimization problem. This allows for adjusting the trade-off between fairness and accuracy based on the specific requirements of the application. Additionally, since  $f(\Omega)$  is not differentiable for a binary classifier, we use a differentiable relaxation based on the classifier's output probabilities rather than decisions.In each iteration, the algorithm computes the consistent upper bound CUB of the classifier's unfairness using the proposed CRA framework, considering the availability of the external data source  $\mathcal{A}_{\Omega}$ .

#### Algorithm 1 Training a fair ML model from biased data

**Input:** Biased training dataset  $D_{\Delta}^{fr}$ , auxiliary information  $I_{\Omega}$ , fairness query f, unfairness threshold  $\tau$ , penalty coefficient  $\lambda$ , learning rate  $\eta$ . **Output:** Fair ML model  $h_{\theta}$  with parameter  $\theta$ .

- 1:  $h_{\theta} \leftarrow \text{random initialization}$
- 2: while not converged do
- 3:  $loss \leftarrow empirical\_loss(h, D_{\Lambda}^{tr})$
- 4: CUB  $\leftarrow$  CRA $(h_{\theta}, f, D_{\Lambda}^{tr}, I_{\Omega})$
- ▶ results in Section 3.1
- 5:  $loss \leftarrow loss + \lambda \cdot max\{CUB, \tau\}$
- 6:  $\theta \leftarrow \theta \eta \cdot \text{gradient}(loss, \theta)$

#### 4 FAIRNESS AND SELECTION BIAS

In this section, we examine the relationship between selection bias and the fairness of predictive models. Specifically, we determine conditions under which selection bias may lead to unfair classifiers. To isolate the impact of selection bias from other sources of bias, such as bias due to finite data or bias due to ML model itself, we first define a fair data distribution that is free from any biases.

Definition 4.1 (Fair data distribution). A data distribution  $\Omega$  that is compatible with a causal diagram G is considered fair for learning a classifier to predict outcome Y if any Bayes-consistent ML model trained on a sufficiently large sample from  $\Omega$  results in a fair classifier h. Conversely, if this is not the case, the data distribution is considered unfair for learning Y.

We now establish a graphical criterion on the data collection diagram  $\mathcal{G}$  such that training a classifier on the data suffering from selection bias leads to an unfair classifier.

PROPOSITION 4.1. A data distribution faithful to a data collection diagram G is unfair for learning a classifier that predicts outcome Y iff either (1) the original data distribution compatible with G is unfair for learning a classifier, or (2) the following conditions C1 - C2 hold.

- C1 The outcome variable  $Y \in Pa(C)$ .
- C2 The selection variable C is either a child of the protected attribute S or there exists a variable  $X \in X \setminus A$  such that the selection variable  $X \in Pa(C)$  and there is an open path between X and S that is not closed after conditioning on A.

Example 4.2. We illustrate with data collection diagrams in Figure 3. When the set of admissible variables is empty  $(A = \emptyset)$ , only the diagram in Figure 3c introduces unfairness; corresponding to the condition in Proposition 4.1, which we have proven causes unfairness to an otherwise fair distribution. Specifically, as the original data distribution is compatible with G and fair, case (1) in Proposition 4.1 is not satisfied for all graphs. As for the conditions in case (2), only Figure 3c and 3d satisfy C1. Between these two structures, only Figure 3c satisfies C2, as it requires another parent  $X \in X \setminus A$ . In Figure 3c, we can set  $X = X_2$ , which becomes the child of Y after selection (C1) and is dependent on S in the original data distribution when conditioned on  $A = \emptyset$  (C2).

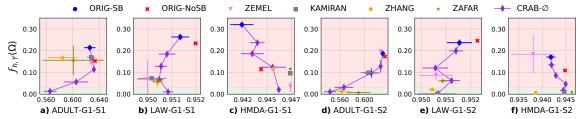
An important conclusion from this section is that when discrimination arises only from selection bias, the selection of data points is influenced by the training label Y. This leads to the violation of the independence assumption  $Y \not\perp\!\!\!\perp C \mid X$ . However, most techniques in ML for addressing selection bias assume that selection bias leads to covariate shift, i.e.,  $Y \perp\!\!\!\perp C \mid X$ . Hence, these techniques cannot handle fairness issues stemming from selection bias. In contrast, CRAB can handle situations where the selection bias is label-dependent, offering solutions not provided by other methods.

#### 5 EXPERIMENTS

We assess how selection bias impacts classifier fairness and demonstrate the efficacy of Crab in achieving fairness while preserving accurate predictions. The code and input files used to generate the results can be found at [1]. We aim to address the following questions. Q1: Can Crab leverage varied amounts of external data to guarantee fairness in the presence of selection bias? How does it compare to the state-of-the-art fair classification methods and current techniques in handling selection bias? (Section 5.2). Q2: When

**Table 3:** Average runtime in seconds for CRAB and baselines. Results of REZAEI on HMDA are omitted due to the huge runtime of the used KDE method while applying on large data (did not finish in 100hrs, expected time required >1 year).

Dataset	Att. [#]	Rows[#]	Orig	Zemel	Kamiran	Zhang	Zafar	Cortes	REZAEI	Crab-Ø	Crab-Suff	Crab-NoA	Crab-NoU
Adult	8	45k	0.8s	56.5s	4.6s	5.6s	1.1s	4.6s	8m5s	1.8s	2.5s	2.7s	5.3s
Law	11	18k	0.6s	43.3s	11s	3.7s	0.8s	10.9s	3m17s	3.4s	5.7s	6.1s	12s
HMDA	8	3.2m	6.7s	45m32s	9m	1m57s	8.9s	8m57s	DNF	16.8s	4m5s	4m8s	8m45s
Syn	5	200k	1.8s	4m36s	56.5s	23.2s	3.1s	56.5s	37m24s	15.6s	28.5s	28.7s	59.4s



**Figure 4:** Equality of opportunity (y-axis)-F1-Score (x-axis) comparison for Crab- $\emptyset$  and baseline methods in the absence of external data. (a to c) correspond to scenario 1, where  $f_{h,Y}(\Delta)$  largely deviates from  $f_{h,Y}(\Omega)$ ; (d to f) correspond to scenario 2, where  $f_{h,Y}(\Delta) \approx f_{h,Y}(\Omega)$ . Selection variable C is placed as a child of S and another attribute  $X \in X$  (same as Figure 3a).

does selection bias introduce unfairness in scenarios where the unbiased data generative process is fair? **Q3**: How does CRAB adapt to other fairness metrics and classification techniques? (Section 5.3)

# 5.1 Setup

**Datasets.** We used the following datasets: **Adult** contains financial and demographic data to predict if an individual's income exceeds \$50K, with gender as the protected attribute. **Law** contains law school student data to predict exam outcome (Pass/Fail), with race as the protected attribute. **HMDA** contains mortgage application data to predict loan approval or denial, with race as the protected attribute. **Syn** is synthetic data generated based on Figure 3.

We introduced selection bias in our datasets by simulating six different mechanisms, by adding the selection variable as a child of various attribute sets and varying the selection probability. This bias was applied to both the training and validation sets, while the test data remained untouched. We repeated the experiments five times and report the average and standard deviation for each method. Our evaluation of the techniques includes statistical parity and equality of opportunity as the fairness measures. A lower value of  $f(\Omega)$  and a higher F1-Score indicate better performance.

ML Models. We evaluate with logistic regression (LR), Support Vector Machine (SVM) with a linear kernel, and neural network (NN) models, implemented using PyTorch [56]. Logistic regression was used as the default classifier unless otherwise specified.

**External Data Sources.** We evaluated CRAB under the following settings of access to external data. (1) CRAB-Ø, with no external data to use Proposition 3.3; (2) CRAB-SUFF, with sufficient external data to apply Proposition 3.4; (3) CRAB-NOU, with limited external data to utilize Proposition 3.7; and (4) CRAB-NOA, with no external data about admissible attributes to apply Proposition 3.8.

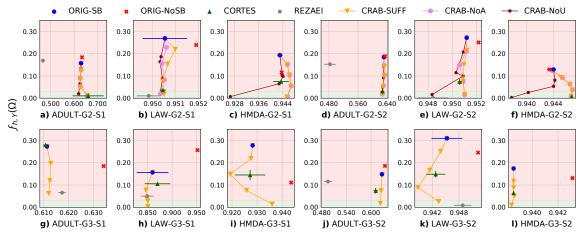
Baselines. We compare Crab with the following representative baselines: Zemel [81] and Kamiran [38] are pre-processing methods that modify the training data to obscure information about the protected attributes. Zhang [82] and Zafar [79, 80] are inprocessing methods that maximize model quality while minimizing fairness violation. Cortes is an inverse propensity score

weighting (IPW) based pre-processing method that utilizes external data to recover from selection bias [19]. Rezaei is an inprocessing method that minimizes the worst-case unfairness with respect to equality of opportunity for Logistic Regression [60]. Cortes and Rezaei are techniques used to address dataset shift from domain adaptation, and are applied only in the presence of external data. We utilized the IBM's AI Fairness 360 [9] implementation for the pre-processing methods and the original implementation of Rezaei [59]. Other baseline methods were implemented from scratch in our framework to ensure a fair comparison.

# 5.2 Solution Quality and Fairness Comparison

The performance of Crab and the baselines was evaluated for varying levels of access to external data. In all figures, the region with a green background indicates that the fairness requirements were satisfied, while the red region indicates an unfair region.

5.2.1 Absence of external data. We compared CRAB-∅ with the fair ML baselines. We varied the fairness requirement  $\tau$  from 0 to  $\theta$ , where  $\theta$  was the unfairness of the classifier trained on the biased dataset. To simulate two different scenarios, we introduced the selection node C as a child of S and another variable  $X_2$ , and varied the selection criterion,  $Pr_{\Omega}(C = 1 \mid Pa(C))$ . In the first scenario (S1), the biased dataset had a much lower level of unfairness compared to the test data, making it necessary to consider the selection bias to ensure fairness over the test data. In contrast, the biased dataset was approximately as fair as the unbiased test data in the second scenario (S2), making it possible to enforce fairness on the biased training data to ensure fairness on the unbiased test data. Solution quality. Figure 4 compares the fairness of the trained classifier and F1-Score. We observe that CRAB-Ø learns a fair classifier  $(\int_{h Y}(\Omega) < 0.05 \text{ for } \tau = 0 \text{ corresponding to the point with }$ lowest y-coordinate) for all datasets across both scenarios. In fact, CRAB-Ø achieves perfect fairness while incurring a very low loss in F1-Score for the Law and HMDA datasets. In contrast, most of the baseline methods fail to completely remove the model's discrimination in most of the cases. In certain cases, baseline techniques like ZEMEL improve fairness (HMDA-G1-S1), but the same technique returns a highly unfair classifier for HMDA-G1-S2. This is



**Figure 5:** Equality of Opportunity (y-axis)-F1-Score (x-axis) comparison for Crab-Suff, Crab-NoU, Crab-NoA, Cortes, and Rezaei given the existence of external data. (a to f) have a selection variable similar to Figure 3b, and (g to l) have a selection variable similar to Figure 3c. S1 corresponds to the scenario where  $f_{h,Y}(\Delta)$  largely deviates from  $f_{h,Y}(\Omega)$ , and S2 denotes the scenario where  $f_{h,Y}(\Delta) \approx f_{h,Y}(\Omega)$ . Error Bars for Crab-Suff, Crab-NoA and Crab-NoU are omitted for clarity. Results of Rezaei on HMDA are omitted due to huge runtime.

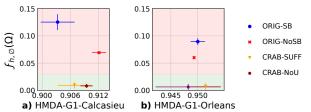
because the selection mechanism creates a false sense of fairness on training data ( $f_{h,Y}(\Delta) \approx 0$ ), while  $f_{h,Y}(\Omega)$  remains high. This is the reason that most of the baselines achieve worse fairness for the first scenario compared to the second one. It shows that the two scenarios behave differently even though the selection bias is a function of the same set of attributes (causal diagram does not change), with the only difference being the selection probabilities. However, Crab-0 upper bounds  $f_{h,Y}(\Omega)$ , which helps to enforce fairness across both settings for all datasets.

Key Takeaway. Crab- $\emptyset$  (with fairness requirement threshold  $\tau=0$ ) achieves perfect fairness for all scenarios and datasets, while baseline methods demonstrate aberrant behavior.

Comparison between ORIG-NoSB and CRAB-0. Figure 4 demonstrates that CRAB-0 outperforms ORIG-NoSB in fairness whenever it achieves an F1-score comparable to that of ORIG-NoSB (red point has higher equality of opportunity compared to CRAB-0 point at same x-coordinate). The only case where ORIG-NoSB has marginally higher accuracy than CRAB-0 is Law-G1-S2 with an insignificant accuracy difference (< 1%). This highlights that CRAB-0 is capable of achieving an F1-score comparable to that of ORIG-NoSB even without external data to address selection bias.

Quality vs. fairness tradeoff. Unlike most prior fair ML techniques, Crab allows the user to specify a fairness requirement  $\tau$ , which allows to simulate varying needs. Figure 4 shows that Crab- $\emptyset$  achieves the same fairness and accuracy as Orig-SB when  $\tau$  exceeds the fairness bound in Theorem 3.3. On reducing  $\tau$ , Crab- $\emptyset$ 's fairness improves consistently with a minor or no loss in F1-score until  $\tau > 0.1$ . Further reducing  $\tau$  to 0 worsens the F1-score for the Adult dataset by around 8% but less than 1% for all other datasets. We observe that Crab- $\emptyset$  and baseline techniques achieve similar F1-scores when Crab- $\emptyset$  is configured to achieve similar fairness. This demonstrates Crab- $\emptyset$  ability to match baseline performance while achieving perfect fairness by achieving zero  $\int_{b} \gamma(\Omega)$ .

*Key Takeaway.* Crab-0 considerably improves fairness of the trained classifier, with only a minor loss in F1-Score.



**Figure 6:** Statistical Parity (y-axis)-F1-Score plots (x-axis) for two parishes in Louisiana where CRAB-SUFF and CRAB-NOU use real-world census to provide external information for training.

5.2.2 Availability of external data. In this experiment, we assess the performance of CRAB under different levels of availability of external data. We consider three different cases. (1) Sufficient information: this setting is applicable when external data source about some statistics of the unbiased distribution are available. CRAB-SUFF uses estimates of  $Pr_{\mathcal{O}}(\boldsymbol{u} \mid s, \boldsymbol{a})$  for model training. (2) **Missing** A: this setting is applicable when external data source about A is not available. For example, when A is the prediction target  $A = \{Y\}$ , we cannot access labels for the unbiased distribution. (3) **Missing** *U*: this setting considers availability of partial external data source about a set of attributes U. For a fair comparison, we compare CRAB under these settings with baselines that use external data (Cortes and Rezaei). Note that Cortes requires additional information compared to CRAB-SUFF, which relies only on the estimates of  $Pr_{\Omega}(u \mid s, a)$  computed from unbiased external data. In contrast, Rezaei requires unlabeled external data, which is similar to the setting of CRAB-NoA.

**Case Study: Louisiana.** We examine the effects of selection bias in the HMDA dataset for two parishes in Louisiana: Calcasieu and Orleans. Using public census data [73] as the source of external data, we ran Crab-Suff, Crab-NoA, and Crab-NoU with  $\tau$  set to 0. Selection bias was introduced based on age and race. Despite the real-world ratios not being exactly consistent, Crab successfully trains a fair model (Figure 6). Furthermore, the F1-Score of the trained model is higher than that of Orig-SB in Calcasieu. This experiment demonstrates the potential of Crab to use public data to

train fair ML models. Next, we compare the performance of CRAB with other baselines and varied settings of external data.

**Solution quality.** The comparison between Crab and other baselines, including Cortes, Rezaei, and Orig-NoSB, is shown in Figure 5. The results demonstrate that all CRAB methods attain approximately zero equality of opportunity for  $\tau = 0$ , with only minimal reductions in F1-Score. In fact, the quality of the fairest classifier is higher than the fairness-agnostic classifier trained on the original data (HMDA-G2-S2 and HMDA-G3-S1). We tested the fairness-accuracy tradeoff in detail in Figure 7 and show that ensuring fairness can indeed improve overall classifier performance. In contrast, Cortes and Rezaei baselines use additional external data to recover from selection bias but remain unstable with respect to fairness. Cortes helps to ensure fairness in Figure 5 (a to d), but the trained classifier is highly unfair in Figure 5 (g to l). Cortes relies on the estimation of propensity scores, which are highly dependent on the quality of classifiers learned to estimate  $Pr_{\Omega}(C=1 \mid X=x)$  and  $Pr_{\Omega}(C=1 \mid Y=1, X=x)$ . Noisy estimation of these probabilities affects Cortes performance. Similarly, Rezaei fails to ensure fairness on many cases due to inaccurate density ratio estimation. Note that both CRAB-NoA and REZAEI require unlabeled external data, while CRAB-NoA outperforms REZAEI in terms of both accuracy and stability (Figure 5 (a to f)).

CRAB-NoU is a bound-based approach, meaning a loose upper bound for  $f_{h,Y}(\Omega)$  could result in excessive fairness restrictions. The superiority of bound-based or estimation-based approaches cannot be determined universally. However, if the difference between the upper bound and  $f_{h,Y}(\Omega)$  is significant, using a boundbased approach like CRAB-NoU may negatively impact classifier performance to ensure fairness, as shown in Figure 5 (f).

Effect of varying external data. Out of three CRAB methods, CRAB-NoU requires the least external data, followed by CRAB-NoA and Crab-Suff. However, these two methods are not shown in Figure 5 (g-l) (Graph G3) as they are not suitable for this scenario. In Figures 5 (a-f), both CRAB-SUFF and CRAB-NoA achieve slightly higher F1-Scores for the same level of fairness compared to CRAB-NoU. Crab-NoU eliminates bias by setting a cap on  $\int_{h,Y}(\Omega)$  instead of estimating it, which may lead to a non-monotonic skyline as seen in Law-G2-S2. Nonetheless, CRAB ensures fairness across all scenarios regardless of the level of access to external data.

Key Takeaway. Crab-Suff produces the most accurate model with zero equality of opportunity. Despite Cortes utilizing more external data and REZAEI having the same level of external data access as CRAB-NoA, these methods do not consistently result in a fair classifier.

Running time. The running time comparison between CRAB, ORIG and all baselines with the logistic regression classifier is presented in Table 3. The results show that CRAB takes less than 10 seconds for small datasets (Adult and Law), and under 5 minutes for larger datasets like HMDA (except CRAB-NoU which is slightly slower). Among the baselines, Rezaei had the longest running time, taking over 24hrs for the HMDA dataset. Although faster than Rezaei, Zemel still took about 10× longer than Crab. The other baselines had a similar execution time as CRAB.

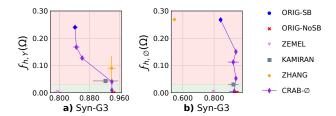


Figure 7: A Discrimination (y-axis)-F1-Score (x-axis) comparison for CRAB-0 and baseline methods evaluated on Syn. ZAFAR was omitted due to its extremely low F1-Score in this experiment.

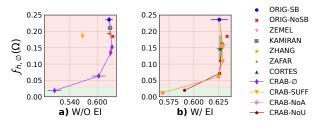


Figure 8: Statistical Parity (y-axis)-F1-Score (x-axis) comparison for CRAB and baseline methods for Adult-G1-S1.

In this section, we examine how CRAB and other baselines respond

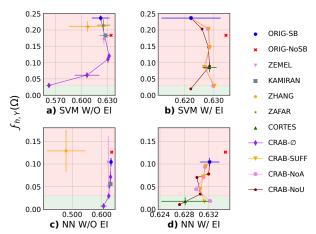
# **Sensitivity to Parameters**

to changes in fairness metric, ML model, and external data size. Fairness metric. Figure 8 compares CRAB on both settings, for the Adult dataset and for statistical parity. In figure 8(a), all baselines achieve statistical parity of 0.15 except CRAB-0, which achieves zero statistical parity. Further, we observe that CRAB-Ø achieves the maximum F1-Score of 0.63 while achieving a statistical parity of less than all other techniques, including the classifier trained

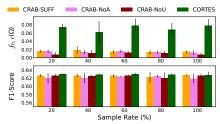
on the unbiased original dataset. This demonstrates the effectiveness of CRAB-Ø at achieving fairness even without any external data. Figure 8 compares CRAB under three different settings of access to external data (CRAB-NOA, CRAB-NOU, CRAB-SUFF). We observe that Crab (with  $\tau$  is set to zero) achieves statistical parity  $\approx$ 0 across all settings. Further, the Cortes baseline does not achieve fairness even though it has access to unbiased data. By comparing figures 8(a) and (b), it can be seen that when external data is used, CRAB-NoU is able to achieve an F1-Score of 0.59 with zero statistical parity, as opposed to 0.52 without external data. Additionally, both CRAB-NoA and CRAB-SUFF reach an F1-Score above 0.57. This demonstrates that incorporating any level of external data can improve the performance of the trained classifier.

ML Models. CRAB can be adapted to a variety of classification algorithms by modifying its loss function. Figure 9 shows that CRAB produces fair results with both SVM and NN classifiers, while most of the baseline models still display unfairness ( $f_{h Y}(\Omega) > 0.05$ ). Further, CRAB-Ø achieves the maximum F1-Score with the maximum fairness ( $f_{h,Y}(\Omega) < 0.12$  in Figure 9(a)), while all baselines perform worse ( $f_{h,Y}(\Omega) > 0.17$ ). Comparing Figure 9 (a) and (b), we observe that CRAB-SUFF achieves higher F1-Scores than CRAB- $\emptyset$  while maintaining  $f_{h,Y}(\Omega) < 0.03$ . We observe similar trends for the neural network (Figure 9 (c), (d)).

Size of external data. We evaluated the impact of varying external data size on the performance of CRAB (with  $\tau = 0.01$ ) and



**Figure 9:** Equality of Opportunity (y-axis)-F1-Score (x-axis) comparison for CRAB and baseline methods on the Adult dataset with the selection mechanism described in Figure 3b.



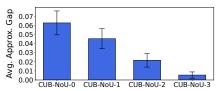
**Figure 10:** The upper plot shows the Equality of Opportunity-Sample Rate; the lower plot shows the F1-Score-Sample Rate.

baselines by using a randomly chosen subset of the unbiased training data as an external data source. Figure 10 shows that all techniques have similar F1-Scores, however, Crab has a considerably lower  $f_{h,Y}(\Omega)$  compared to Cortes. In fact, Crab achieved the desired level of fairness for all sampling rates and access to external data. On the other hand, Cortes showed the highest standard deviation of unfairness, indicating instability in its performance. As the sample size decreases, the standard deviation of Crab methods increases slightly, indicating a slight decrease in stability as the quality of estimated ratios degrades.

**Tightness of CUBs.** In this experiment, we analyze how the CUBs employed by Crab-NoU (Proposition 3.7) differ based on the availability of various levels of external data sources, namely varying sizes of U'. We trained 100 classifiers with Crab- $\emptyset$  on the Adult data with randomly injected selection bias (|U|=3), and compare the average gaps between  $\int_{h,Y}(\Omega)$  and CUBs computed with different sizes of U'. Our results in Figure 11 demonstrate that increasing the size of U' leads to tighter CUBs. This observation empirically validates the benefits of incorporating more variables into the external data source. Although CUB-NoU-0 has a large approximation gap, it still enables us to train a fair and accurate model, as demonstrated in previous experiments, e.g. Figure 4. Note that in the absence of any external data source, Crab-NoU boils down to Crab- $\emptyset$ , corresponding to CUB-NoU-0. Likewise, when U' = U, Crab-NoU is equivalent to Crab-Suff, which uses CUB-NoU-3.

# 6 RELATED WORK

Query answering in the presence of selection bias has been studied in the data management community [54, 55]. Also, the effect of



**Figure 11:** Average Approximation Gap comparison for Crab-NoU with different levels of external data source. CUB-NoU-n refers to the CUB obtained in Proposition 3.7 when |U'| = n.

incomplete and representation bias in data and its impact on fairness has been studied in databases [36, 66]. In the context of machine learning, selection bias has also been studied for its impact on models' fairness [14, 23, 30, 47, 75]. The most related studies are [30, 75], which analyze how selection bias can affect fairness guarantees. Our study rigorously investigates the impact of selection bias on fairness and provides conditions for learning fair models in the presence of selection bias with varying external data sources. Selection bias has also been studied in dataset shift research, including covariate shift, prior probability shift, and concept shift [50]. Reweighing is a common solution but can lead to errors [19, 44]. Other studies have explored conditions for accuracy and fairness under distribution shift in general [14, 17, 46] or learning a fair model under covariate shift [23, 60]. The impact of concept shift on fairness in performance prediction has also been investigated [48]. Some works have proposed causal methods to build fair models based on data collection information, but only address covariate shift and require target distribution knowledge [67, 70]. Under the assumption that the unbiased data distribution is within the proximity of the biased training distribution, [47] analyzes the robustness of decision trees; [64, 87] provide theoretical bounds on fairness in the target distribution using samples from the target distribution; [72] and [74] train ML models that are fair on any distribution near the training distribution using distributionally robust optimization. Our work addresses all types of dataset shifts resulting from selection bias. For a comprehensive overview of related work, please refer to the extended version [89].

### 7 CONCLUSIONS

In this paper, we proposed a novel framework for ensuring fairness in machine learning models trained from biased data. Our framework, inspired by data management principles, presents a method for certifying and ensuring the fairness of predictive models in scenarios where selection bias is present. This framework only requires understanding of the data collection process and can be implemented regardless of the information available from external data sources. Our findings show our framework's success in learning fair models, while accounting for biases in data. It serves as a valuable tool for practitioners striving for fairness in areas often dealing with inherent selection bias.

# 8 ACKNOWLEDGMENTS

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