

# Explain3D: Explaining Disagreements in Disjoint Datasets

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## ABSTRACT

Data plays an important role in applications, analytic processes, and many aspects of human activity. As data grows in size and complexity, we are met with an imperative need for tools that promote understanding and explanations over data-related operations. Data management research on explanations has focused on the assumption that data resides in a single dataset, under one common schema. But the reality of today’s data is that it is frequently un-integrated, coming from different sources with different schemas. When different datasets provide different answers to semantically similar questions, understanding the reasons for the discrepancies is challenging and cannot be handled by the existing single-dataset solutions.

In this paper, we propose explain3D, a framework for explaining the disagreements across disjoint datasets (3D). Explain3D focuses on identifying the reasons for the differences in the results of two semantically similar queries operating on two datasets with potentially different schemas. Our framework leverages the queries to perform a semantic mapping across the relevant parts of their provenance; discrepancies in this mapping point to causes of the queries’ differences. Exploiting the queries gives explain3D an edge over traditional schema matching and record linkage techniques, which are query-agnostic. Our work makes the following contributions: (1) We formalize the problem of deriving optimal explanations for the differences of the results of semantically similar queries over disjoint datasets. Our optimization problem considers two types of explanations, provenance-based and value-based, defined over an evidence mapping, which makes our solution interpretable. (2) We design a 3-stage framework for solving the optimal explanation problem. (3) We develop a smart-partitioning optimizer that improves the efficiency of the framework by orders of magnitude. (4) We experiment with real-world and synthetic data to demonstrate that explain3D can derive precise explanations efficiently, and is superior to alternative methods based on integration techniques and single-dataset explanation frameworks.

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## 1. INTRODUCTION

Data drives modern applications, analytic processes, and business decisions, heavily influencing many aspects of human activity: from product recommendations and friend connections, to autonomous vehicle decisions and election campaign strategies. Understanding data and the results of processes that operate on data becomes critical in promoting trust in data-driven decisions and in facilitating debugging and repair of errors [52]. Even within the relatively simple setting of relational data and queries, the explosive data sizes, source heterogeneity, and issues of poor data quality make providing explanations a challenging problem.

Existing data management solutions that aim to provide explanations for query results [44, 45, 56] have an important limitation: They focus on a single dataset, where data conforms to a single common schema. However, modern data rarely conforms to this integrated ideal. More often than not, datasets evolve separately, under different schemas, and even datasets from trustworthy sources frequently end up diverging, both in format and content, causing headaches to downstream applications and users. For example, Open Data [40], released by governments and organizations, is typically of high quality, publicly available, and freely used and distributed. Such datasets may be related and overlapping, but their separate production and evolution lead to disagreements that can cause confusion to users and incorrect analyses.

**EXAMPLE 1 (ACADEMIC DATA DISAGREEMENT).** *We collect two publicly-available academic datasets: the UMass-Amherst dataset on undergraduate programs<sup>1</sup>, and the National Center for Education Statistics (NCES) dataset<sup>2</sup>. Both data sources are reputable and contain high-quality information. Nevertheless, querying both datasets for the number of undergraduate degree programs at UMass Amherst yields vastly different answers.*

	<i>UMass-Amherst data (<math>D_{UMass}</math>)</i>	<i>NCES data (<math>D_{NCES}</math>)</i>
<b>Schema:</b>	<i>Major(Major, Degree, School)</i>	<i>School(ID, Univ_name, City, Url) Stats(ID, Program, bach_degr)</i>
<b>Query:</b>	$Q_1$ : SELECT COUNT(Major) FROM Major;	$Q_2$ :SELECT SUM(bach_degr) FROM School, Stats WHERE Name = 'UMass-Amherst' AND School.ID=Stats.ID;
<b>Answer:</b>	113	90

*Existing explanation solutions can only be applied with respect to one of these datasets at a time, by asking questions such as “Why is the result of  $Q_1$  (resp.  $Q_2$ ) high (resp. low)?” But these would not provide meaningful explanations in this case, as each tuple contributes the same to the aggregate of  $Q_1$ , and prioritizing tuples with low bach\_degr in the provenance of  $Q_2$  would be arbitrary, not grounded on the actual differences with  $Q_1$ .*

<sup>1</sup><https://www.umass.edu/gateway/academics/undergraduate>

<sup>2</sup><https://nces.ed.gov>: A open dataset presented in simplified schema.

SQL query $Q_1$ :		SQL query $Q_2$ :		SQL query $Q_3$ :		SQL query $Q_4$ :	
SELECT COUNT(program)		SELECT COUNT(Major)		SELECT SUM(Num_bach)		SELECT SUM(Num_major)	
FROM $D_1$ ;		FROM $D_2$ WHERE Univ='A';		FROM $D_3$ ;		FROM $D_4$ ;	
Dataset $D_1$ :		Dataset $D_2$ :		Dataset $D_3$ :		Dataset $D_4$ :	
Program	Degree	Univ	Major	College	Num_bach	Campus	Num_major
Accounting	B.S.	A	Accounting	Business	2	South campus	1
CS	B.A.	A	CSE	Engineering	2	North campus	2
CS	B.S.	A	ECE	Computer Science	1	East campus	1
ECE	B.S.	A	EE				
EE	B.S.	A	Management				
Management	B.A.	A	Design				
Design	B.A.	B	Art				
(a) $Q_1(D_1) = 7$		(b) $Q_2(D_2) = 6$		(c) $Q_3(D_3) = 5$		(d) $Q_4(D_4) = 4$	

**Figure 1: Four queries, operating on disjoint datasets, for answering the question: How many undergraduate degree programs are provided by University A? However, all queries yield different answers:  $Q_1(D_1) = 7$ ,  $Q_2(D_2) = 6$ ,  $Q_3(D_3) = 5$ , and  $Q_4(D_4) = 4$ .**

Example 1 illustrates the predicament of dealing with disagreements in disjoint datasets and how single-dataset explanation frameworks fall short. Attempting to use data cleaning [10, 36, 42] and data fusion [9, 21] techniques towards this problem meets similar challenges. These techniques attempt to reconcile the datasets, but are agnostic to the queries of interest, which may very well be contributors to the discrepancy. Ultimately, our goal is not to reconcile the differences between two datasets and consolidate them, but rather to explain the reasons of disagreement between two queries on those datasets, whose results are expected to be the same.

In this paper, we introduce `explain3D`,<sup>3</sup> a framework for deriving interpretable explanations for the disagreement in the results of two semantically similar queries.<sup>4</sup> `explain3D` leverages the queries in coordination with existing schema matching and entity resolution methods to derive a semantic mapping across the relevant parts of the queries’ provenance. It processes this initial mapping to find optimal provenance-based (mismatched tuples) and value-based (mismatched values) explanations and summarizes these explanations to increase understandability. For the disagreement in Example 1, `explain3D` finds that (1) several tuples in  $D_{UMass}$  (such as majors “Equine Management” and “Turfgrass Management”) do not correspond to tuples in  $D_{NCES}$ , and (2) there is a mismatch of contributions for some tuples—for example, “Computer Science” is counted twice in  $Q_1$  for the distinct B.S. and B.A. degrees, but “Computer Science” has `bach_degr=1` in  $D_{NCES}$ . `explain3D` further analyzes the common properties of the derived explanations to summarize them as: (1) There is a large portion of mismatches for majors with `Degree=“Associate degree”` in  $D_{UMass}$ ; (2) There are majors with multiple degree types in  $D_{UMass}$ , counted multiple times by  $Q_1$ , for which `bach_degr=1` in  $D_{NCES}$ .

`explain3D` addresses the following challenges:

**Different schemas.** Data sources often adopt different schemas and may thus store their data with different granularities. For example, in Example 1,  $D_{UMass}$  lists each degree program as an individual tuple whereas  $D_{NCES}$  stores an aggregate of the degrees in each program in the attribute `bach_degr`. Such differences significantly increase the difficulty in determining the mapping relationship between tuples in different datasets.

**Missing data mapping.** Data mapping or tuple mapping is essential in deriving the explanations. However, existing record linkage and entity resolution techniques [5, 37] typically target mapping entities within the same dataset or datasets with highly similar

schemas. In contrast, in our setting, we can leverage the queries to provide us both with the relevant provenance, and clues of the matching attributes.

**Distinct queries.** Two queries meant to retrieve the same information across two datasets with different schemas are bound to be structurally different. Differences in the queries are confounded with differences in the data and schemas, obscuring the causes of discrepancies and making deriving explanations more challenging.

We make the following contributions.

- We introduce the necessary modeling abstractions and formalize the problem of deriving optimal explanations for the disagreements between the results of two semantically similar queries over two disjoint datasets. We identify explanations as one of two types: provenance-based (indicating mismatched tuples between the two datasets) and value-based (indicating incorrect values in particular tuples). These explanations are defined over an *evidence mapping*, which is an explanation of the explanations themselves, making our method interpretable. (Section 2)
- We introduce `explain3D` as a 3-stage framework for solving our optimal explanation problem. The first stage leverages the queries and standard schema matching and record linkage methods to derive an initial mapping between the relevant provenance data. The second stage, which is the core of our approach, models the optimization problem as a mixed integer linear program (MILP) and produces a refined *evidence mapping*. This mapping, informed by the queries and the datasets, pinpoints the discrepancies between the two datasets. The third stage relies on standard methods to analyze the common properties of the discrepancies and summarize the explanations. (Section 3)
- We propose a smart-partitioning optimizer that breaks the optimization problem of `explain3D`’s second stage into smaller components, which can be solved separately, increasing the efficiency and scalability of our framework. (Section 4)
- We perform extensive experimental evaluation of `explain3D` using real-world and synthetic data, comparing it with a state-of-the-art single-dataset explanation framework, state-of-the-art entity resolution approaches, and multiple baselines. Our evaluation shows that `explain3D` is superior in explanation accuracy compared to the alternatives, and the smart-partitioning optimizer is robust to multiple parameter settings and increases efficiency by orders of magnitude with little to no loss of accuracy. (Section 5)

## 2. EXPLANATIONS FOR DISJOINT DATA

In this section, we use a running example inspired by Example 1 to introduce our concepts and abstractions for modeling explanations for disagreements in disjoint datasets.

<sup>3</sup>Pronounced “explained”

<sup>4</sup>In the context of our work, semantic similarity is subjectively determined by human raters, and assumed as part of the input. This is analogous to the standards of semantic similarity in the natural language processing literature [19].

EXAMPLE 2. Figure 1 displays four semantically similar queries that answer the question “How many undergraduate programs are provided by University A?” The queries compute the same thing semantically, but they operate on different datasets, with different schemas:  $D_1$  lists the undergraduate programs at University A and  $Q_1$  counts them;  $D_2$  lists the majors at multiple universities and  $Q_2$  selects the ones from University A and counts them;  $D_3$  lists the number of bachelor degrees per college at University A and  $Q_3$  sums them;  $D_4$  lists the number of majors per campus at University A and  $Q_4$  sums them. While all four queries are correct semantically, they ultimately yield different results.

Manually, one can easily contrast  $Q_1$  and  $Q_2$ . The Program and Major attributes are a direct match, and each program in  $D_1$  corresponds to a major in  $D_2$  and vice versa, through a one-to-one mapping: ‘Accounting’ to ‘Accounting’, ‘CS’ to ‘CSE’, ‘ECE’ to ‘ECE’, etc. This reveals that computer science is counted twice in  $Q_1$ , for the B.S. and B.A. degrees, but only once in  $Q_2$ , which explains the difference in their results. Moreover, the mapping of tuples between the two datasets is an interpretable explanation (evidence) of the explanation itself.

The correspondence between  $Q_1$  and  $Q_3$  is a little less straightforward, because the data is stored at different granularities (list of programs vs aggregates per college). However, the queries are still comparable. The Program attribute semantically maps to the College attribute in a containment relationship: each program typically corresponds to a college—Accounting and Management are part of the Business School, ECE and EE are part of the College of Engineering, and CS is part of the College of Computer Science. This mapping reveals that (1) CS is counted twice in  $Q_1$ , for the B.S. and B.A. degrees, but  $D_3$  only lists one bachelor degree in the CS College, and (2) the Design program is missing from  $D_3$ .

While we can reason about the differences of  $Q_1$ ,  $Q_2$ , and  $Q_3$ , we cannot compare them with  $Q_4$  because the Campus attribute does not meaningfully correspond in a direct or containment relationship with the other datasets.

This example highlights several concepts: (1) attribute matches and their implications to (2) comparability of queries, (3) explanations as mismatched tuples or mismatched values, and (4) evidence mappings that support the derived explanations. We proceed to formalize these concepts and define the problem of deriving explanations for disagreements in the results of semantically similar queries over disjoint datasets.

## 2.1 Problem input: Queries, data, and matches

In this paper, we focus on queries of the general relational algebra form  $Q = \pi_o \sigma_C(X)$ , where  $X$  can be a single relation or an arbitrary query, allowing joins, unions, and subqueries;  $C$  also allows any operators, except UDFs. We restrict the projection,  $o$ , to be either a set of attributes,  $o = A \subseteq \text{attr}(X)$ , or one of the five main SQL aggregate functions (SUM, COUNT, AVERAGE, MAX, MIN),  $o = \text{aggr}(A_i), A_i \in \text{attr}(X)$ . Compared to prior work on explanations over a single database [13, 45, 49, 56], which mostly focus on flattened queries in select-project-join (SPJ) and select-project-join-aggregate (SPJA) format, our framework supports a wider range of queries.

As Example 2 showed, some queries are not comparable ( $Q_1$  and  $Q_4$ ). Reasoning about these cases would require external information, not derivable by standard matching and linking methods. We cannot derive explanations for these cases—this appears impossible without external information—and we focus on comparable queries. As Example 2 highlighted, comparability is determined by semantic mappings that match attributes of the queries. We formalize these *attribute matches* below.

Notation	Description
$Q = \Pi_o \sigma_C(R)$	A query over relation $R$ in database $D$ .
$\mathcal{M}_{\text{attr}}(Q_1, Q_2) = (A_i \phi A_j)$	Attribute matches.
$\mathcal{M}_{\text{tuple}} = \{(t_i, t_j, p), \dots\}$	Tuple matches.
$P(A_1, \dots, A_k, I)$ or $P$	The provenance relation of query $Q$ .
$T$	Canonical tuples of query $Q$ .
$t.I$	The impact of a tuple $t$ .
$E = (\Delta, \delta   \mathcal{M}_{\text{tuple}}^*)$	Explanations and their evidence.
$\Delta = \{t, \dots\} \in E$	Provenance-based explanations.
$\delta = \{t.I \mapsto t.I^*\} \in E$	Value-based explanations.
$\mathcal{M}_{\text{tuple}}^* \subseteq \mathcal{M}_{\text{tuple}}$	Evidence of a set of explanations.

Figure 2: Summary of notations.

DEFINITION 2.1 (ATTRIBUTE MATCHES). Given two queries  $Q_1$  over relation  $R_1$  and  $Q_2$  over relation  $R_2$ , we represent the semantic mapping among their attributes as **attribute matches**, denoted with the matching function  $\mathcal{M}_{\text{attr}}$ :

$$\mathcal{M}_{\text{attr}}(Q_1, Q_2) = (A_i \phi A_j)$$

where  $A_i, A_j$  are sets of categorical attributes in  $R_1$  and  $R_2$ , respectively, and  $\phi \in \{\equiv, \sqsubseteq, \supseteq\}$  is the semantic relation between two sets of attributes [26].

In our definition of matching attributes, we borrow the notion of the semantic relation  $\phi$  from prior work [26]. A set of attributes  $A_i$  can be *semantically equivalent* to  $A_j$  ( $A_i \equiv A_j$ ), corresponding to a one-to-one mapping between instantiations of  $A_i$  and  $A_j$ , *less general* than  $A_j$  ( $A_i \sqsubseteq A_j$ ), corresponding to a many-to-one mapping, or *more general* than  $A_j$  ( $A_i \supseteq A_j$ ), corresponding to one-to-many mapping. Note that semantic equivalence does not imply a condition on cardinality and two semantically equivalent sets of attributes may in fact have arbitrary overlap. For example, the sets  $A_i = (\text{address}, \text{city}, \text{state}, \text{zip})$  in  $R_1$  and  $A_j = (\text{address})$  in  $R_2$  can be semantically equivalent ( $A_i \equiv A_j$ ). In our running example,  $\mathcal{M}_{\text{attr}}(Q_1, Q_2) = (\text{program}) \equiv (\text{major})$ , and  $\mathcal{M}_{\text{attr}}(Q_1, Q_3) = (\text{program}) \sqsubseteq (\text{college})$ . The attribute matches can be derived from standard schema matching techniques [2, 6, 26, 57]. Deriving these matches is not a focus in our work, and we treat them as part of our input.

One can consolidate or separate matches over sets of attributes, e.g.,  $(\text{zip}, \text{city}) \sqsubseteq (\text{county})$  becomes  $(\text{zip}) \sqsubseteq (\text{county})$  and  $(\text{city}) \sqsubseteq (\text{county})$ . Our framework applies to both cases. From here on, for ease of exposition, we will assume that the attribute matches are on a single attribute from each relation, and we will simply denote them with  $\mathcal{M}_{\text{tuple}}$  when the queries are clear from the context.

If there exists at least one attribute match between two queries, we can derive explanations for their differences (comparable queries); otherwise, the queries are not comparable ( $Q_1$  and  $Q_4$ ).

DEFINITION 2.2 (COMPARABLE QUERIES). Two queries  $Q_1$  over relation  $R_1$  and  $Q_2$  over relation  $R_2$  are comparable if and only if  $\mathcal{M}_{\text{attr}}(Q_1, Q_2) \neq \emptyset$ .

We focus on comparable queries in this work, and from here on we will assume that the queries we discuss are comparable. To derive explanations for query disagreements, we need to analyze the contents of the two datasets and reason about their correspondence. We do not need to do so for the entire datasets, but rather for the parts that contribute to the queries (provenance). For example, in  $Q_2$  only the tuples in  $D_3$  with Univ='A' are part of the provenance. To facilitate exposition, we derive a *provenance relation*.

DEFINITION 2.3 (PROVENANCE RELATION). Given a query  $Q = \pi_o \sigma_C(R)$  over relation  $R(A_1, \dots)$ , we derive a **provenance**

**relation**  $P(A_1, \dots, I)$  as follows: For each tuple  $t \in \sigma_c(R)$ , we create a tuple  $t' = (t, I)$  in  $P$ , where  $t'.I = \Pi_{o'}(t)$ , with  $o' = 1$  if  $Q$  is a non-aggregate query, and  $o' = o$  otherwise. The impact of a tuple measures its statistical contribution to the result of query  $Q$ .

In our running example, the provenance relation of  $Q_1$  has 7 tuples (same as  $D_1$ ), each with impact 1; the provenance relation of  $Q_3$  has 3 tuples (same as  $D_3$ ), with impacts 2, 2, and 1, same as the corresponding values of the Num\_bach attribute.

Given two queries, the tuples of their provenance relations can be associated through mappings such as the ones described in Example 2. We formalize the tuple mapping below.

**DEFINITION 2.4 (TUPLE MAPPING).** Given relations  $R_1$  and  $R_2$ , the **tuple mapping** between  $R_1$  and  $R_2$  is a set of tuple matches:

$$\mathcal{M}_{tuple} = \{(t_i, t_j, p), \dots\}$$

where  $t_i \in R_1$  and  $t_j \in R_2$  are two tuples, and  $p \in (0, 1]$  is the probability that tuple  $t_i$  and tuple  $t_j$  correspond to the same or associated (with respect to containment) entities.

In Example 2, a possible tuple mapping between  $Q_1$  and  $Q_2$  can be (omitting superfluous attributes for simplicity)  $\mathcal{M}_{tuple} = \{(\text{Accounting}, \text{Accounting}, 1.0), (\text{CS}, \text{CSE}, 0.9), (\text{ECE}, \text{ECE}, 1.0), (\text{EE}, \text{EE}, 1.0), (\text{Management}, \text{Management}, 1.0), (\text{Design}, \text{Design}, 1.0)\}$ . Deriving such matches can be done with traditional record linkage techniques [5, 8, 16, 20, 54]. We use such techniques as blackbox components in our framework to derive an initial tuple mapping. This initial mapping is typically crude, with many possible tuple matches of varied probabilities, and it needs to be refined into the correct mapping  $\mathcal{M}_{tuple}^*$ . This refinement is a core part of our framework, which we will discuss in Section 3.

## 2.2 Problem output: The explanations

Example 2 highlighted the two generic types of explanations we derive: (1) provenance-based explanations, indicating mismatched tuples between the two datasets, and (2) value-based explanations, indicating incorrect values or contributions for particular tuples. We formalize these explanations below.

**DEFINITION 2.5 (EXPLANATIONS).** Given two queries  $Q_1$  and  $Q_2$  and their provenance relations  $P_1$  and  $P_2$ , the explanations of their differences include two generic types:

- A **provenance-based explanation** is a tuple  $t \in P_1$  (resp.  $t \in P_2$ ) such that  $t$  does not map to a  $t' \in P_2$  (resp.  $t' \in P_1$ ). We use  $\Delta$  to denote a set of provenance-based explanations.
- A **value-based explanation** specifies an impact value change,  $t.I \mapsto t.I^*$ , for a tuple  $t \in P_1 \cup P_2$ , meaning that  $t$  should have impact  $t.I^*$  rather than  $t.I$ . We use  $\delta$  to denote a set of value-based explanations.

Example 2 highlights a provenance-based explanation for the disagreement of  $Q_1$  and  $Q_3$  (the Design program is missing from  $D_3$ ), and a value-based explanation ( $D_3$  only lists one bachelor degree in the CS College, when it should be two). The derived explanations are tightly coupled with the tuple mapping. In comparing  $Q_2$  and  $Q_3$ , a mapping that matches CSE with the Computer Science College, will produce different explanations than a mapping that matches CSE to the College of Engineering. Typically, the initial mappings ( $\mathcal{M}_{tuple}$ ) derived from standard entity resolution and linkage techniques are probabilistic, and would assign the two possible matches for CSE with two distinct probabilities. Our goal is to discover the right mapping  $\mathcal{M}_{tuple}^*$  leading to the correct (optimal) set of explanations; we call this refined mapping the *evidence*

mapping (or evidence for short). The evidence mapping is a subset of the initial mapping ( $\mathcal{M}_{tuple}^* \subseteq \mathcal{M}_{tuple}$ ), and needs to conform to certain properties discussed in Section 3.

The final product of our framework is a set of explanations and their evidence, reported as  $E = (\Delta, \delta | \mathcal{M}_{tuple}^*)$ . The evidence  $\mathcal{M}_{tuple}^*$  is an explanation of the explanations themselves, making our result fully interpretable.

## 2.3 Optimal explanations for 3D

We now define the problem of deriving optimal explanations for disagreements in disjoint data, which we will refer to as EXP-3D.

**PROBLEM 1 (THE EXP-3D PROBLEM).** Given two queries  $Q_1$  and  $Q_2$  with provenance relations  $P_1$  and  $P_2$ , respectively, and a set of initial tuple matches  $\mathcal{M}_{tuple}$ , our goal is derive a set of explanations,  $E = (\Delta, \delta | \mathcal{M}_{tuple}^*)$  that **maximize the probability**:

$$Pr(E | P_1, P_2, \mathcal{M}_{tuple})$$

More informally, we are looking for the set of explanations and their evidence mapping that are the most likely, given the queries' provenance and the initial probabilistic tuple mapping. In our running example, suppose that the initial mapping for  $Q_2$  and  $Q_3$  assigns two possible matches for CSE, Computer Science and Engineering, each with some probability. This indicates two possible cases for  $\mathcal{M}_{tuple}^*$ , mapping CSE to Computer Science in one case and Engineering in the other. The former choice results in a single provenance-based explanation (the tuple with major='Design' in  $D_2$  does not have a match in  $D_3$ ). The latter choice, results in the same explanation and, in addition, that the tuple with College='Computer Science' in  $D_3$  does not have a match in  $D_2$ , and that the Num\_bach value of the Engineering tuple in  $D_3$  is wrong. Clearly, the former choice is better. Intuitively, a particular tuple mapping identifies specific discrepancies, which we map to explanations, and fewer discrepancies are typically preferred.

In Section 3.1, we analyze the calculation of the objective function of Problem 1, and reduce it to a simpler scoring function that is both tractable and theoretically-grounded. In Section 3.2, we describe a framework for deriving the explanations and evidence mapping through a translation to Mixed Integer Linear Programs (MILP). Then, in Section 3, we describe a smart-partitioning optimizer that improves the efficiency of our basic approach by several orders of magnitude.

## 3. DERIVING EXPLANATIONS

In this section, we present explain3D, a 3-stage framework that solves Problem 1. The first stage (Section 3.1) refines the provenance data into a canonical form that is easier to analyze. With data in this canonical form, we define essential properties for evidence mappings and explanations, and use them to simplify the objective function of Problem 1. The second stage (Section 3.2), which is the core of our approach, models the optimization problem as a mixed integer linear program (MILP) and produces a refined *evidence mapping* and the corresponding explanations. The third stage (Section 3.3) relies on standard methods to analyze the common properties of the discrepancies and summarize the explanations.

### 3.1 Stage 1: Canonicalization and Simplification

The provenance relation  $P_1$  of  $Q_1$  has two tuples for the CS program, one for the B.S. and one for the B.A. degree. The degree information is not relevant to the comparison with  $Q_2$ , and it is not part of the mapping between  $Q_1$  and  $Q_2$ . Thus the two CS tuples in  $P_1$  are indistinguishable with respect their role in the disagreement between  $Q_1$  and  $Q_2$ . This indicates that the provenance relation

contains redundancy. We consolidate redundant tuples and their impact through *canonicalization*. Canonicalization groups tuples with the same values for the matching attributes and sums their impacts. Canonicalization does not change the provenance relations of queries that require a strict one-to-one mapping (queries with AVG/MAX/MIN aggregation). The canonical relation of  $Q_1$  has 6 tuples (instead of 7 in  $P_1$ ), and CS is represented by a single tuple with impact 2 (Figure 3a).

**DEFINITION 3.1 (CANONICAL RELATION).** *Given a provenance relation  $P$  of a query  $Q$ , and attribute matches  $\mathcal{M}_{attr}$ , the canonical relation  $T$  of  $P$  is derived with the query:*

$$T = \pi_{\mathcal{A}, I}(\mathcal{A}\mathcal{G}_{SUM(I)}(P))$$

Where  $\mathcal{A}$  is a set of matching attributes that appear in  $\mathcal{M}_{attr}$ ;  $\mathcal{A}\mathcal{G}_{SUM(I)}$  is the Group By operation over attributes  $\mathcal{A}$  with aggregate function  $SUM$  on the impact attribute  $I$ .

**EXAMPLE 3.** *Figure 3 shows the canonical relations of  $Q_1$  and  $Q_2$  based on the attribute matches  $\mathcal{M}_{attr} = (\text{program} \equiv \text{major})$ . The canonical relation of  $Q_1$  is constructed with the query:*  
 SELECT program, COUNT(I) AS I FROM  $P_1$  GROUP BY program

Canonicalization simplifies the datasets without losing information necessary for the reasoning on disagreements. It further allows us to identify and formalize essential properties for explanations and evidence mapping, which we analyze next.

### Explanation properties

**Completeness.** Explanations define refinements on the canonical relations. A provenance-based explanation indicates the removal of tuples, and a value-based explanation modifies a tuple’s impact. Our goal is to identify a set of explanations that is *complete*: if one performs all the refinements defined by the explanations, the queries would return the same result. We evaluate completeness through the properties of valid mapping and equal impact. In the following, we denote  $T_1^* = \delta(T_1 \setminus \Delta)$  and  $T_2^* = \delta(T_2 \setminus \Delta)$  as the refined tuples of the canonical relations.

**Mapping validity.** The attribute matches ( $\mathcal{M}_{attr}$ ) between two queries imply the cardinality of the tuple matches between the two canonical relations. If two attributes have an equivalence match, e.g.,  $\text{program} \equiv \text{major}$ , then the canonical relations should have a one-to-one mapping of their tuples. Thus, in Figure 3, each tuple in  $T_1$  should map to one tuple in  $T_2$ . If it is a less general match, e.g.,  $\text{program} \sqsubseteq \text{college}$ , then the mapping should be many-to-one (many programs map to one college). We can never have many-to-many mappings.

Initial tuple mapping, however, typically do not conform to the required cardinality, as they frequently assign several probabilistic matches for each tuple. For example, the CSE major in  $Q_2$  may be mapped to two colleges in  $Q_3$ , Engineering and Computer Science, which violates the many-to-one cardinality requirement for two relations. Our goal is to produce a refined mapping  $\mathcal{M}_{tuple}^*$  that conforms to the cardinality requirements of the attribute matches  $\mathcal{M}_{tuple}^*$ ; we call such a mapping *valid*.

**DEFINITION 3.2 (VALID MAPPING).** *Given attribute matches  $\mathcal{M}_{attr} = (\mathcal{A}_i \phi \mathcal{A}_j)$ , and two sets of refined tuples,  $T_1^*$  and  $T_2^*$ , the mapping  $\mathcal{M}_{tuple}^*$  is *valid* if and only if the following are true:*

- If  $\mathcal{A}_i \sqsubseteq \mathcal{A}_j$ , then  $\forall t \in T_1^*, |\{t|(t, t', p) \in \mathcal{M}_{tuple}^*\}| \leq 1$
- If  $\mathcal{A}_i \supseteq \mathcal{A}_j$ , then  $\forall t \in T_2^*, |\{t|(t', t, p) \in \mathcal{M}_{tuple}^*\}| \leq 1$
- If  $\mathcal{A}_i \equiv \mathcal{A}_j$ , then both the above conditions hold.

rowID	Program	I	rowID	Major	I
$p_1$	Accounting	1	$m_1$	Accounting	1
$p_2$	CS	2	$m_2$	CSE	2
$p_3$	ECE	1	$m_3$	ECE	1
$p_4$	EE	1	$m_4$	EE	1
$p_5$	Management	1	$m_5$	Management	1
$p_6$	Design	1	$m_6$	Design	1

(a)  $T_1$ : Canonical relation for  $Q_1$  (b)  $T_2$ : Canonical relation for  $Q_2$

**Figure 3: Canonical relations for queries  $Q_1$  and  $Q_2$  of Figure 1.  $I$  denotes the impact of the tuples.**

**Impact equality.** Tuples of the canonical relations and their mapping form a bipartite graph. In a valid mapping, where the matches can only be one-to-one, one-to-many, or many-to-one, the graph separates into connected components. Each component contains the tuples that correspond to each other semantically. When the two query results agree, the total impact on each side of the bipartite graph is the same within each connected component. Thus, our goal is to find a set of explanations, such that the refined canonical relations  $T_1^*$  and  $T_2^*$  demonstrate such impact equality.

**DEFINITION 3.3 (IMPACT EQUALITY).** *Given canonical relations  $T_1^*$  and  $T_2^*$ , and a bipartite graph  $G$  formed by a valid mapping  $\mathcal{M}_{tuple}^*$  between  $T_1^*$  and  $T_2^*$ , the impact equality property is satisfied if and only if for all connected components  $(T_1', T_2')$  of  $G$ :*

$$\sum_{t \in T_1'} (t.I) = \sum_{t \in T_2'} (t.I)$$

**DEFINITION 3.4 (COMPLETE EXPLANATIONS).** *A set of explanations  $E = (\Delta, \delta|\mathcal{M}_{tuple}^*)$  over canonical relations  $T_1$  and  $T_2$  is *complete* if  $\mathcal{M}_{tuple}^*$  is a valid mapping and  $T_1^* = \delta(T_1 \setminus \Delta)$  and  $T_2^* = \delta(T_2 \setminus \Delta)$  satisfy the impact equality property.*

### Explanation problem revisited

The objective function of Problem 1 maximizes the probability  $Pr(E|P_1, P_2, \mathcal{M}_{tuple})$ . This probability can be equally and more efficiently computed over the canonical relations, which are a (lossless, for the purposes of this problem) summary of the provenance relations:  $Pr(E|P_1, P_2, \mathcal{M}_{tuple}) = Pr(E|T_1, T_2, \mathcal{M}_{tuple})$ .

From Bayesian inference, this is proportional to the product of three probabilities:

$$Pr(E|T_1, T_2, \mathcal{M}_{tuple}) \propto Pr(T_1, T_2|E)Pr(\mathcal{M}_{tuple}|T_1, T_2, E)Pr(E) \quad (1)$$

We next consider each of the three probabilities separately.

$Pr(T_1, T_2|E)$ . Assuming that tuples are independent, we have:

$$Pr(T_1, T_2|E) = \prod_{t \in T_1 \cup T_2} Pr(t|E) \quad (2)$$

We use  $\alpha$  and  $\beta$  to denote the a priori probabilities that  $t \in T_1 \cap T_2$  and that  $t$  has correct impact  $t.I$ , respectively. Intuitively,  $\alpha, \beta \in (0.5, 1]$ , as a tuple is more likely to be covered by both queries and have correct impact than not.<sup>5</sup> We then compute the probabilities of the different cases of  $t$ ’s inclusion in a set of explanations  $E$  as:

$$\begin{aligned} Pr(t|t \notin \Delta, t \notin \delta) &= \alpha\beta; & Pr(t|t \notin \Delta, t \in \delta) &= \alpha(1 - \beta); \\ Pr(t|t \in \Delta, t \notin \delta) &= 1 - \alpha; & Pr(t|t \in \Delta, t \in \delta) &= 0. \end{aligned} \quad (3)$$

$Pr(T_1, T_2|E)$  is then derived from Equations (2)-(3). Larger  $\Delta$  and  $\delta$  lead to lower probabilities, thus the computation prioritizes smaller provenance- and value-based explanations.

<sup>5</sup>For simplicity, we assume the same  $\alpha$  and  $\beta$  for all tuples, but our framework can handle different values across tuples.

$\Pr(\mathcal{M}_{tuple} | \mathbf{T}_1, \mathbf{T}_2, \mathbf{E})$ . Assuming independence in tuple matches:

$$\Pr(\mathcal{M}_{tuple} | T_1, T_2, E) = \prod_{m \in \mathcal{M}_{tuple}} \Pr(m | T_1, T_2, E) \quad (4)$$

In addition, for a tuple match  $m = (t_i, t_j, p)$ , the probability that tuples  $t_i$  and  $t_j$  match is  $p$ , thus:

$$\begin{aligned} \Pr(m | m \in \mathcal{M}_{tuple}^*, t_i, t_j \in T_1 \cup T_2) &= p; \\ \Pr(m | m \notin \mathcal{M}_{tuple}^*, t_i, t_j \in T_1 \cup T_2) &= 1 - p; \\ \Pr(m | t_i, t_j \notin T_1 \cup T_2) &= 0. \end{aligned} \quad (5)$$

$\Pr(\mathcal{M}_{tuple} | T_1, T_2, E)$  is then derived from Equations (4)-(5).

The probability computation prioritizes tuple matches with higher probabilities in the evidence mapping.

**Pr(E).** In this paper, we simply set the prior probability of a set of explanations  $E$ , based on whether it is complete (Definition 3.4). If  $E$  is complete, then  $\Pr(E) = 1$ ; otherwise,  $\Pr(E) = 0$ . These priors force our framework to only consider explanations that resolve all disagreements.

We can then compute the objective function from Equation (1). In practice, to improve efficiency we calculate and later optimize the probability in the logarithmic space:

$$\begin{aligned} \log(\Pr(E | T_1, T_2, \mathcal{M}_{tuple})) &\propto \\ \log(\Pr(T_1, T_2 | E)) &+ \log(\Pr(\mathcal{M}_{tuple} | T_1, T_2, E)). \end{aligned} \quad (6)$$

Through a reduction from the Exact Cover problem<sup>6</sup>, we can prove that Problem 1 is NP-complete [53].

**THEOREM 3.5.** EXP-3D (Problem 1) is NP-complete.

## 3.2 Stage 2: MILP transformation

In this section, we show how stage 2 of explain3D transforms the EXP-3D problem into a mixed integer linear program (MILP). This transformation allows explain3D to use modern constrained optimization solvers to derive the optimal explanations. Later, in Section 4, we show how to optimize computation in this stage, through a smart-partitioning optimizer.

To translate an instance of the EXP-3D problem into a MILP problem, we first convert tuples, their tuple matches, and the associated explanations into linear constraints; we then express the explanation completeness properties, using linear constraints; we complete the translation process by formalizing a linear expression for the probability of the explanations.

### Expressing explanations

To express the explanations, we first introduce a binary variable for each tuple  $t_i \in T_1 \cup T_2$  and a binary variable for each tuple match  $(t_i, t_j, p)$ ; we then translate the changes suggested by the explanations into linear constraints.

**Tuple:** Given a tuple  $t_i = (t_i.A_1, \dots, I)$ , there are two types of explanations that may be associated with this tuple: (1) a provenance-based explanation ( $t_i \in \Delta$ ); (2) a value-based explanation ( $t_i \in \delta$ ). We use a binary variable  $x_i$  to indicate whether tuple  $t_i$  is included in an provenance-based explanation; To express the value-based explanation, we use an integer variable  $t.I^*$  for tuple  $t$ 's refined impact and a binary variable  $y_i$  representing whether the tuple's refined

impact is the same as its original impact ( $y_i = 1$ ) or not ( $y_i = 0$ ). The binary variable  $y_i$  should satisfy the following constraint.

$$y_i = (t.I^* = t.I) \quad (7)$$

When  $x_i = 1$ , the tuple  $t_i \in \Delta$  is selected as a provenance-based explanation; when  $x_i = 0$ , the tuple  $t_i$  remains in the canonical relation and its impact is set to  $t.I^*$ .

Based on the binary variables and Equation (3), we express the probability of the explanations being associated with tuple  $t_i$  as:

$$\log(\Pr(t_i)) = x_i \otimes a + \underline{(1 - x_i) \otimes ((1 - y_i) \otimes b + y_i \otimes c)}$$

In the above expression,  $\otimes$  represents regular multiplication; we prefer to use  $\otimes$  to indicate that it is the semi-module multiplication by scalars;  $a = \log(1 - \alpha)$ ,  $b = \log(\alpha) + \log(\beta)$ , and  $c = \log(\alpha) + \log(1 - \beta)$  as three constant values. Note that the above Equation is quadratic due to the underlined expression:  $P_i = (1 - x_i) \otimes ((1 - y_i) \otimes b + y_i \otimes c)$ . We linearize  $P_i$ , with the help of two constant numbers  $L$  and  $U$  as follows [3].

$$\begin{aligned} P_i &\geq L \otimes (1 - x_i) \\ P_i &\leq U \otimes (1 - x_i) \\ P_i &\geq (1 - y_i) \otimes b + y_i \otimes c - U \otimes x_i \\ P_i &\leq (1 - y_i) \otimes b + y_i \otimes c - L \otimes x_i \\ \log(\Pr(t_i)) &= x_i \otimes p_1 + P_i \end{aligned} \quad (8)$$

The constant number  $L$  (or  $H$ ) cannot be greater than the lower bound (or smaller than the upper bound) of  $P_i$ .

**Tuple Match:** Given a tuple match  $m = (t_i, t_j, p)$ , we use a binary variable  $z_{i,j}$  to express whether it is a true match: When  $z_{i,j} = 1$ , we include it in the evidence mapping. The probability of this match is computed as follows.

$$\begin{aligned} z_{i,j} &\leq (1 - x_i); \quad z_{i,j} \leq (1 - x_j) \\ \log(\Pr(m)) &= z_{i,j} \otimes \log(p) + (1 - z_{i,j}) \otimes \log(1 - p) \end{aligned} \quad (9)$$

Where  $x_i$  and  $x_j$  are the binary variables associated with  $t_i$  and  $t_j$ , respectively.

### Expressing explanation completeness

We use the explanation variables to express the mapping validity and impact equality properties as linear constraints.

**Valid Mapping:** As required by Definition 3.2, the refined tuple matches  $\mathcal{M}_{tuple}^*$  should follow the valid mapping property, which essentially restricts the degree for some of the tuples to be less than or equal to 1. If  $t_i$  is such a tuple, then we add the constraints:

$$\sum_{(t_i, t_j, p) \in M} z_{i,j} \leq 1 \quad (10)$$

**Equal Impact:** Valid mappings between the canonical tuples  $T_1$  and  $T_2$  can never have many-to-many cardinality. Therefore, in the bipartite graph between  $T_1$  and  $T_2$  under a valid mapping, at least one of  $T_1$  or  $T_2$  is guaranteed to have only tuples with maximum degree of 1. This observation allows us to simplify the specification of the connected components in the bipartite graph and the corresponding impact calculations. Suppose that all tuples in  $T_1$  have maximum degree 1. Then the set of connected components is:

$$\mathcal{S} = \{(\eta(t_j), t_j, M) | t_j \in T_2\}$$

where  $\eta(t_j)$  is the set of  $T_1$  tuples that are adjacent to  $t_j \in T_2$ . Consider one connected component  $(\eta(t_j), t_j, M) \in \mathcal{S}$ , the total impact of  $T_1$  in the component is  $I_l = \sum_{t_i \in \eta(t_j)} z_{i,j} \otimes t_i.I^*$ ; and

<sup>6</sup>The Exact Cover problem is one of Karp's 21 NP-complete problems [30].

---

**Algorithm 1:** The basic solution

---

**Input** : Two sets of canonical tuples  $(T_1, T_2)$  and acquired tuple matches  $(\mathcal{M}_{tuple})$   
**Output**: A set of explanations

- 1  $milp\_vars, milp\_cond, prob\_expr \leftarrow \emptyset$ ;
- 2 **foreach** tuple  $t$  in  $T_1 \cup T_2$  **do**
- 3      $milp\_vars \leftarrow milp\_vars \cup \text{DefineTupleVariables}(t)$ ;
- 4      $milp\_cond \leftarrow milp\_cond \cup \text{TupleImpactCondition}(t)$ ;
- 5      $prob\_expr \leftarrow prob\_expr \cup \text{TupleProbability}(t)$ ;
- 6 **foreach** mapping  $m$  in  $\mathcal{M}$  **do**
- 7      $milp\_vars \leftarrow milp\_vars \cup \text{DefineMappingVariables}(m)$ ;
- 8      $prob\_expr \leftarrow prob\_expr \cup \text{MappingProbability}(m)$ ;
- 9  $milp\_cond \leftarrow milp\_cond \cup \text{FormConditions}(milp\_vars)$ ;
- 10  $milp \leftarrow \text{FormMILP}(milp\_cond, prob\_expr)$ ;
- 11  $solved\_vars \leftarrow \text{SolveMILP}(milp)$ ;
- 12  $E \leftarrow \text{DecodeVariables}(solved\_vars)$ ;
- 13 **return**  $E$ ;

---

the total impact of  $T_2$  tuples is  $I_r = t_j \cdot I^*$ . Here, we linearize the quadratic equation  $I_l$  using the same method as Equation (8).

$$\begin{aligned} I_i &\leq U \otimes z_{i,j} \\ I_i &\geq L \otimes z_{i,j} \\ I_i &\leq t_i \cdot I^* - L \otimes (1 - z_{i,j}) \\ I_i &\geq t_i \cdot I^* - U \otimes (1 - z_{i,j}) \end{aligned} \quad (11)$$

Where  $I_i = z_{i,j} \otimes t_i \cdot I^*$  is an element in  $I_l$ ;  $L$  and  $U$  are two constants that cannot be greater than the lower bound (or smaller than the upper bound) of a tuple’s impact.

Finally, the equal impact property requires:

$$\sum_{t_i \in \eta(t_j)} I_i = I_l \quad (12)$$

### Formalizing the objective function

The EXP-3D problem aims to derive a set of complete explanations such that the probability of the explanations is maximized. The MILP formulation creates variables for all provenance-based (tuples) and all value-based (impact) explanations. Our objective function can be formulated as a linear expression over the explanation variables in a fashion similar to the constraints of the explanation properties:

$$\log(\text{Pr}(E|\mathcal{T}, \mathcal{M})) = \sum_{t \in \mathcal{T}} \log(\text{Pr}(t)) + \sum_{m \in \mathcal{M}} \log(\text{Pr}(m)) \quad (13)$$

Where  $\mathcal{T} = T_1 \cup T_2$ ,  $\mathcal{M} = \mathcal{M}_{tuple}$ ;  $\log(\text{Pr}(t))$  and  $\log(\text{Pr}(m))$  are formulated by Equation (8) and Equation (9) respectively.

### The algorithm

Algorithm 1 provides the pseudocode implementing the MILP transformation described in this section. The algorithm first iterates over all tuples in the input to define variables, construct constraints, and express the tuple probabilities in Lines 3-5. The algorithm then iterates over all the tuple matches and formalizes the probability expression in Line 8 according to Equation (9). Next, the algorithm constructs constraints for the completeness requirement, as in Equations (10)-(12), by a *FormConditions* function (Line 9). With the variables and constraints, the algorithm completes the MILP problem formulation and calls a MILP solver to get a solution (Line 10-11). We derive the final explanations from the MILP solution by including an explanation or evidence (tuple match) if the solve value of the corresponding binary variable is 1 (Line 12).

## 3.3 Stage 3: Summarization

The product of stage 2 of explain3D is a set of explanations and their evidence mapping. But when the discrepancies between two datasets are extensive, the derived explanations could involve a large number of tuples and values. Reviewing such explanations can be tedious. Stage 3 of our framework is tasked with summarizing and abstracting the explanations to reduce their size and increase their understandability. As in Example 1, we may use Degree= “Associate degree” in  $DUMass$  to summarize the common patterns of the derived explanations, which is easier to understand than presenting the explanations individually.

Different summarization methods are possible. Explain3D marks tuples associated with explanations as a “target” and then uses existing techniques, such as Data Auditor [27] and Data X-Ray [50] to identify common patterns for the target tuples. Alternatively, “target” tuples could be treated as examples by QBE (Query-By-Example) techniques [17, 23, 43, 46], which can then generate SQL queries that precisely describe them. Developing novel summarization methods is not a focus of our work in this paper, and thus stage 3 relies on existing tools. Detailed stage 2 explanations are still available through explain3D, for users who prefer to peruse the more precise and detailed causes of the disagreement.

## 4. PARTITIONING OPTIMIZATION

A critical problem with stage 2 of the explain3D framework is that it does not scale for problems with a large number of tuples and tuple matches. The problem is that the generated MILP grows to sizes that can stump even state-of-art solvers. To improve the efficiency of the basic algorithm, we can split the bipartite graph  $G = (T_1, T_2, \mathcal{M}_{tuple})$  into its maximal connected components and solve the problem in each component separately. This method requires linear time,  $O(|T_1| + |T_2| + \mathcal{M}_{tuple})$ , to derive the connected components and it does not sacrifice the accuracy. However, it fails to achieve any efficiency or scalability guarantees, as in the worst case,  $G$  may be connected.

Inspired by the connected components approach, we propose a method to divide the original problem into a collection of sub-problems with bounded sizes such that each sub-problem is guaranteed to be small enough to solve. Our partitioning method is based on the Graph Partitioning Problem (GPP) [11, 31, 34, 41], which aims to minimize the total weight of the edge cuts<sup>7</sup>.

### PROBLEM 2 (THE GRAPH PARTITIONING PROBLEM).

Given a number  $k \in \mathbb{N}_{>1}$ , a bipartite graph  $G = (T_1, T_2, \mathcal{M}_{tuple})$  formed by tuples and their matches, and an upper bound  $L_{max}$  for the maximum partition size, we seek a partition  $\Pi$  of  $T_1 \cup T_2$  with disjoint collections of tuples  $\Pi = \{(T_{1,1}, T_{2,1}), \dots, (T_{1,k}, T_{2,k})\}$  such that:

- $T_{1,1} \cup \dots \cup T_{1,k} = T_1$  and  $T_{2,1} \cup \dots \cup T_{2,k} = T_2$ ;
- $|T_{1,i}| + |T_{2,j}| \leq L_{max}$ ;
- $\text{EdgeCutSum}(\Pi) = \sum_{(t_i, t_j) \in E} w(t_i, t_j)$  is minimized.

Where  $E = \{(t_i, t_j), \dots\}$  denotes the set of edges across partitions;  $w(t_i, t_j)$  denotes the weight of edge  $(t_i, t_j)$ ;  $|T_{1,i}| + |T_{2,j}| \leq L_{max}$  is the balancing constraint over the maximum size of one partition.

In our setting, a naïve way to assign the edge weights is by using the tuple matches’ probabilities:  $w(t_i, t_j) = p$ . However, this setting is ill-suited for our problem: According to our objective function (Problem 1), cutting a high probability tuple match tends to hurt our objective value much more than cutting multiple lower

<sup>7</sup>Edge cuts refer to edges across partitions.

---

**Algorithm 2:** The pre-partitioning algorithm

---

**Input** : A bipartite graph  $G = (T_1, T_2, \mathcal{M}_{tuple})$  and thresholds  $\theta_l, \theta_h, R$   
**Output**: A simplified graph  $G_c = (C_1, C_2, \mathcal{M}_c)$

- 1  $C_1, C_2, \mathcal{M}_c \leftarrow \emptyset$ ;
- 2 **foreach** tuple  $t$  in  $T_1 \cup T_2$  **do**
- 3     **if**  $t.isVisited$  **then**
- 4         | continue
- 5          $(T'_1, T'_2) \leftarrow \text{FindHighProbTuplesDFS}(t, G, \theta_h)$ ;
- 6          $(C'_1, C'_2) \leftarrow \text{MergeTuples}(T'_1, T'_2)$ ;
- 7          $(C_1, C_2) \leftarrow \text{UpdateMergedTuples}(C'_1, C'_2)$ ;
- 8 **foreach** mapping  $(t_i, t_j, p)$  in  $\mathcal{M}_{tuple}$  **do**
- 9     |  $(C'_i, C'_j) \leftarrow \text{FindMergedTuples}(C_1, C_2, t_i, t_j)$ ;
- 10    |  $\mathcal{M}_c \leftarrow \text{UpdateEdgeWeight}(C'_i, C'_j, p, R)$
- 11 **return**  $G_c = (C_1, C_2, \mathcal{M}_c)$ ;

---

probability tuple matches with equal or even higher total probabilities. For example, let us assume that we cut a tuple match, with 0.9 probability, that is part of the optimal explanation ( $\mathcal{M}_{tuple}^*$ ). The objective value,  $Pr(E)$ , would drop by 9 times<sup>8</sup> as the probability of this tuple match,  $Pr(m|m \in \mathcal{M}_{tuple}^*)$ , would change from 0.9 to 0.1. This objective value loss is significantly higher than cutting two tuple matches with lower individual (0.6 each) but higher total probabilities (1.2 in total). The latter case would only lead to a objective value drop by 2.25 times. Based on this observation, we prioritize cutting tuple matches with lower probabilities and avoid cutting tuple matches with high probabilities. We achieve this by adjusting the edge weight assignments as below:

$$w(t_i, t_j) = \begin{cases} p \cdot R, & \text{if } p \geq \theta_h; \\ p/R, & \text{if } p \leq \theta_l; \\ p, & \text{otherwise.} \end{cases}$$

Where  $R \in (1, \infty)$  is a constant for rewarding (or penalizing) high probability (or low probability) tuple matches and  $0 \leq \theta_l < \theta_h \leq 1$  are two thresholds specifying low and high probability tuple matches. In this paper, we set  $\theta_l = 0.1, \theta_h = 0.9, R = 100$ .

Existing graph partitioners, e.g., METIS [41] and hMETIS [31], can be used directly to derive the sub-problems, but they are not efficient when  $R$  is large. To further optimize partitioning efficiency, we employ a *pre-partitioning step* that combines tuples connected by high probability tuple matches. This pre-partitioning step can also be considered as an extra coarsening level on top of the *multi-level graph partitioning algorithms* [32, 33]. Empirically, this step achieves  $200\times$  partition time speedup over graphs with  $10K$  tuples without compromising optimality.

Algorithm 2 presents the pseudocode of the pre-partitioning step. The algorithm iterates over tuples in the bipartite graph in arbitrary order and attempts to merge tuples that are connected by high probability tuple matches as much as possible (Lines 2-7). It then iterates over the remaining tuple matches and updates the edge weights of the merged tuples accordingly (Lines 8-10). This algorithm has linear time complexity:  $O(|T_1| + |T_2| + |\mathcal{M}_{tuple}|)$ .

Finally, Algorithm 3 presents our smart-partitioning method. This algorithm first leverages the pre-partitioning algorithm (Algorithm 2) to generate a much smaller graph (Line 1); it then partitions the smaller graph (Line 2) with a standard graph partitioner; it finally produces the final partitioning  $\Pi$  according to the tuples' assigned partitions (Lines 3-6).

<sup>8</sup>This is based on the assumption that the probabilities of other tuples and tuples matches are not impacted.

---

**Algorithm 3:** The smart-partitioning algorithm

---

**Input** : A bipartite graph  $G = (T_1, T_2, \mathcal{M}_{tuple})$ , thresholds  $\theta_l, \theta_h, R$ , the number of partitions  $k$ , and the maximum partition size  $L_{max}$   
**Output**: A partition  $\Pi$

- 1  $G_c \leftarrow \text{PrePartition}(G, \theta_l, \theta_h, R)$ ;
- 2  $\Pi_c \leftarrow \text{GraphPartitioner}(G_c, k, L_{max})$ ;
- 3  $\Pi \leftarrow \text{InitializeKEmptyPartitions}(k)$ ;
- 4 **foreach**  $(C'_1, C'_2)$  in  $G_c$  **do**
- 5     |  $idx \leftarrow \Pi_c(C'_1, C'_2)$ ;
- 6     |  $\Pi[idx] \leftarrow \text{AddTuples}(C'_1, C'_2)$ ;
- 7 **return**  $\Pi$ ;

---

## 5. EXPERIMENTAL EVALUATION

In this section, we evaluate the effectiveness and efficiency of explain3D using both real-world and synthetic data. In particular, we first compare explain3D with several alternative algorithms over two categories of real-world data (Section 5.2); then, we evaluate the performance and benefit of the smart-partitioning optimization over a series of synthetic datasets with diverse properties (Section 5.3).

### 5.1 Experimental setup

All experiments were performed on  $4 \times 2.77$  GHz machines with 32GB RAM running IBM CPLEX [29] as the MILP solver on MacOS version 10.11.6.

#### 5.1.1 Datasets, queries, and gold standards

We first describe the real-world data used in our evaluation; we describe our synthetic data experiments in Section 5.3.

**Academic datasets.** We collect three publicly available academic datasets, the UMass-Amherst dataset on undergraduate programs and the National Center for Education Statistics (NCES) dataset, described in Example 1, and the OSU dataset on undergraduate programs<sup>9</sup>. We create two pairs of datasets for comparisons: (1) *UMass-Amherst vs. NCES*, described in Example 1, and (2) *OSU vs. NCES*, described in the table below. We evaluate all alternative algorithms with queries that compute *the number of undergraduate programs at UMass Amherst (or OSU, respectively)* on each pair of data.

<i>OSU data</i> ( $D_{OSU}$ )	<i>NCES data</i> ( $D_{NCES}$ )
<i>Major</i> (Major, Degree, Campus, School)	<i>School</i> (ID, Univ_name, City, Url) <i>Stats</i> (ID, Program, bach_degr)
$Q_1$ : SELECT COUNT(Major) FROM Major;	$Q_2$ : SELECT SUM(bach_degr) FROM School, Stats WHERE Name = 'OSU' AND School.ID=Stats.ID;

**Gold Standard:** We manually create the gold standard for the explanations and the evidence mapping on both pairs of data. The datasets, queries, and gold standards are publicly available<sup>10</sup>. Figure 4 shows the detailed statistics of the academic datasets.

**IMDb Datasets.** We retrieve the IMDb data<sup>11</sup>, and use it to create a pair of disjoint datasets, as two views with different schemas over the original data. To simulate the real-world disagreements over disjoint data, we choose a schema design for the first view such that a certain portion of data is lost during the data migration process.<sup>12</sup> We further introduce  $\sim 5\%$  random errors to both

<sup>9</sup><http://undergrad.osu.edu/majors-and-academics/majors>

<sup>10</sup><https://bitbucket.org/xlwang/explain3d>

<sup>11</sup><https://datasets.imdbws.com/>

<sup>12</sup>In  $D_{IMDb1}$ , a movie is associated with a single country and genre.

views with the BART system [1]. We create 10 query templates (listed below), mapped over each view, covering a wide range of query types, including joins, subqueries, non-aggregates, and 5 different aggregate functions. We create 10 instantiations of each template, by selecting a random value for  $year \in [1970, 2003]$  for templates  $Q_1$ – $Q_9$ , and a random value for  $genre$  in  $Q_{10}$ , resulting in a total of 100 different queries.

IMDb View 1 ( $D_{IMDb1}$ )	
Movie ( <i>movie_id</i> , <i>title</i> , <i>release_year</i> , <i>genre</i> , <i>country</i> , <i>runtimes</i> , <i>gross</i> , <i>budget</i> )	
Actor ( <i>actor_id</i> , <i>firstname</i> , <i>lastname</i> , <i>gender</i> , <i>dob</i> )	
Director ( <i>director_id</i> , <i>firstname</i> , <i>lastname</i> , <i>gender</i> , <i>dob</i> )	
MovieDirector ( <i>movie_id</i> , <i>director_id</i> )	MovieActor ( <i>movie_id</i> , <i>actor_id</i> )
IMDb View 2 ( $D_{IMDb2}$ )	
Movie ( <i>m_id</i> , <i>title</i> , <i>release_year</i> )	MovieInfo ( <i>m_id</i> , <i>info_type</i> , <i>info</i> )
Person ( <i>p_id</i> , <i>name</i> , <i>gender</i> , <i>dob</i> )	MoviePerson ( <i>m_id</i> , <i>p_id</i> )
Query templates	
$Q_1$	Return actors who were cast in short movies released in $\langle year \rangle$ .
$Q_2$	Return movies directed by someone born in $\langle year \rangle$ .
$Q_3$	Return the number of comedy movies released in $\langle year \rangle$ .
$Q_4$	Return the number of movies released in the US in $\langle year \rangle$ .
$Q_5$	Return the total gross value for movies released in $\langle year \rangle$ .
$Q_6$	Return the maximum gross value for movies released in $\langle year \rangle$ .
$Q_7$	Return the longest movie released in $\langle year \rangle$ .
$Q_8$	Return the average gross value for movies released in $\langle year \rangle$ .
$Q_9$	Return the average runtime for movies released in $\langle year \rangle$ .
$Q_{10}$	Return actresses who have not starred in any $\langle genre \rangle$ movies.

**Gold Standard:** While creating the two disjoint views, we keep track of the data lost in the first view and record the random errors introduced by BART; these are the optimal explanations of the query disagreements. The optimal evidence mapping can also be easily acquired through the mapping between the views and the original dataset. The detailed statistics of the IMDb datasets are shown in Figure 4.

### 5.1.2 Attribute matches and tuple mapping

**Attribute Matches.** The attribute matches ( $\mathcal{M}_{attr}$ ) for the two real-world datasets are shown in Figure 5.

**Tuple Mapping.** In evaluation, we use a similarity-to-probability method [24, 55] to collect the initial tuple mapping ( $\mathcal{M}_{tuple}$ ). This similarity-to-probability method is a two-step process that generates the tuple matches probabilities from their similarity values: (1) it first divides the tuple matches into  $k$  continuous buckets over the similarity values; (2) in each bucket, it calculates the probability of tuple matches by the ratio of true matches within the current bucket. The true matches can be acquired by labeling a subset of data, or by a known gold standard.

To generate the similarity values, we use token-wise Jaccard similarity for *String attributes*:

$$sim(t_i.A, t_j.A) = \frac{|t_i.A \cap t_j.A|}{|t_i.A \cup t_j.A|}$$

We use normalized Euclidean distance on *numeric attributes*:

$$sim(t_i.A, t_j.A) = \frac{1}{1 + |t_i.A - t_j.A|^2}$$

We finally combine the similarity values over multiple attributes by taking their mean value:

$$sim(t_i, t_j) = \frac{\sum_{A \in \mathcal{M}_{attr}} sim(t_i.A, t_j.A)}{|\mathcal{M}_{attr}|}$$

After computing the pair-wise similarity for tuples in the canonical relations, we generate the initial tuple matches and their probabilities with the above similarity-to-probability method. In particular, we divide the tuple matches into 50 buckets and we use the

Academic datasets					
	# of undergrad majors		# of undergrad majors		
	UMass	NCES	OSU	NCES	
$N/ P / T $	113/113/95	239K/81/81	282/282/206	239K/153/153	
$ \mathcal{M}_{tuple} $	169		607		
$ \mathcal{M}_{tuple}^* $	71		140		
$ E  \rightarrow  E_S $	64 $\rightarrow$ 11		127 $\rightarrow$ 16		
IMDb datasets					
	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$
	$ P _{(IMDb1/IMDb2)}$	1.3K/4.6K	2.8K/3.8K	1.6K/3.1K	2.7K/6.2K
$ \mathcal{M}_{tuple} $	0.6M	0.8M	51K	0.3M	1.1M
$ \mathcal{M}_{tuple}^* $	1271	2768	1601	2756	4231
$ E  \rightarrow  E_S $	3.4K $\rightarrow$ 33	1.1K $\rightarrow$ 23	1.5K $\rightarrow$ 28	3.4K $\rightarrow$ 38	5.5K $\rightarrow$ 43
	$Q_6$	$Q_7$	$Q_8$	$Q_9$	$Q_{10}$
	$ P _{(IMDb1/IMDb2)}$	5.8K/5.9K	10.9K/10.9K	3.4K/3.5K	4.8K/4.9K
$ \mathcal{M}_{tuple} $	0.5M	2.2M	0.2M	0.4M	1.3M
$ \mathcal{M}_{tuple}^* $	5353	6259	2365	3147	5959
$ E  \rightarrow  E_S $	1.3K $\rightarrow$ 19	21.1K $\rightarrow$ 86	2.5K $\rightarrow$ 33	3.9K $\rightarrow$ 40	13.4K $\rightarrow$ 75

**Figure 4: Dataset statistics.**  $N$ ,  $|P|$ ,  $|T|$  are the original data size, the provenance relation size, and the canonical relation size, respectively; the size of the initial tuple mapping is  $|\mathcal{M}_{tuple}|$ ; the sizes of the optimal evidence mapping and the optimal explanations are  $|\mathcal{M}_{tuple}^*|$  and  $|E|$ , respectively.  $|E_S|$  is the size of the explanations after summarizing them with Data X-Ray [50, 51].  $N$  for  $D_{IMDb1}$  and  $D_{IMDb2}$  are 3.7M and 6.8M tuples, respectively, for all IMDb queries. In the IMDb datasets, we show the average numbers over 10 instantiations of each query;  $|P|$ ,  $|T|$  in these datasets are the same, so we report only one.

UMass vs. NCES	OSU vs. NCES
(Major.Major) $\sqsubseteq$ (Stats.Program)	(Major.Major) $\sqsubseteq$ (Stats.Program)
IMDb View 1 vs. IMDb View 2	
(Movie.title, Movie.release_year) $\equiv$ (Movie.title, Movie.release_year)	
(Actor.firstname, Actor.lastname, Actor.gender, Actor.dob)	(Person.name, Person.gender, Person.dob)
(Director.firstname, Director.lastname, Director.gender, Director.dob)	(Person.name, Person.gender, Person.dob)

**Figure 5: Attribute matches for the real-world datasets.**

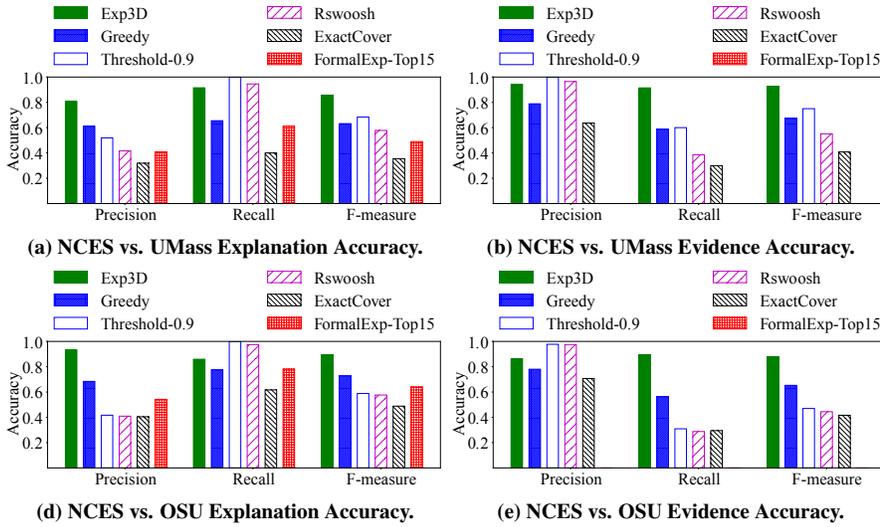
evidence mapping in the gold standard to label a sample of matches and produce the probabilities of the buckets. The sizes of the initial tuple matches for each of the datasets are shown in Figure 4.

### 5.1.3 Algorithms

We compare our framework, explain3D, against FORMAL-EXP, an approach that focuses on explanations in the single dataset setting, RSWOOSH, a state-of-the-art record linkage system, and three additional baseline methods. We describe all the algorithms below.

**FORMALEXP:** FORMAL-EXP explains surprising outcomes of aggregate queries in a single database [45]. To apply FORMAL-EXP in disjoint datasets, we first compare the results of the queries and then ask FORMAL-EXP to explain why the query result is high (or low) on each individual dataset. Tuples that are included by the derived explanations are considered provenance-based explanations. FORMAL-EXP returns the Top- $k$  explanations, and requires  $k$  as an input. In our experiments, we set  $k = 15$ , denoted by FORMAL-EXP-Top15, as it achieves the highest overall accuracy.

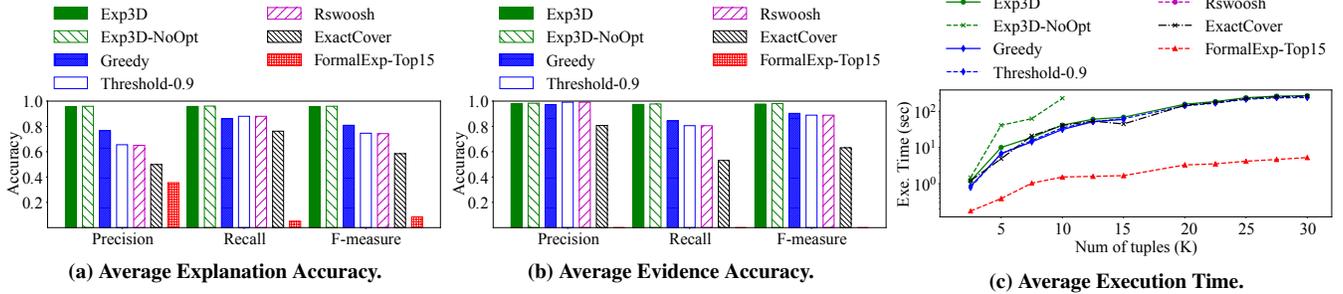
**RSWOOSH:** RSWOOSH [5] is an entity resolution technique that produces deterministic tuple matches. For RSWOOSH, we treat all derived tuple matches as the evidence mapping since their probabilities are all equal to 1.0. We include tuples that do not have a match in this evidence mapping as provenance-based explanations, and tuples with unequal impacts as value-based explanations. Here



Method	NCES/UMass (sec)
FORMALEXP-Top15	0.052
RSWOOSH	0.273
THRESHOLD-0.9	0.276
GREEDY	0.280
EXACTCOVER	0.272
EXPLAIN3D	0.322

Method	NCES/OSU (sec)
FORMALEXP-Top15	0.064
RSWOOSH	0.541
THRESHOLD-0.9	0.581
GREEDY	0.573
EXACTCOVER	0.562
EXPLAIN3D	0.729

**Figure 6: Accuracy and efficiency comparison over Academic datasets. EXPLAIN3D achieves much higher accuracy than the other methods. THRESHOLD obtains high precision but low recall in the derived evidence. FORMALEXP does not provide any tuple matches in the evidence.**



**Figure 7: Accuracy and efficiency comparison over IMDb datasets. EXPLAIN3D achieves near perfect accuracy. RSWOOSH and EXPLAIN3D without the smart-partitioning optimization fails to produce any results for queries with more than 10K tuples in 1hr.**

we use the Jaccard similarity metric to compare string attributes, using 0.75 as the default threshold value.<sup>13</sup>

**THRESHOLD:** THRESHOLD is a simple baseline that refines the initial probabilistic tuple matches by a fixed threshold. It uses the derived evidence mapping to derive explanations, in the same manner as RSWOOSH. In our experiment, we set a threshold of 0.9 and denote it as THRESHOLD-0.9.

**GREEDY:** GREEDY is a baseline that implements explain3D’s objective function (Definition 1), but builds the evidence mapping in a greedy fashion (whereas explain3D derives it by solving constrained optimization problems). Initialized with an empty evidence mapping, GREEDY prioritizes tuple matches with higher probabilities and includes into the evidence the match with highest probability that does not violate the valid mapping restriction (Definition 3.2) and improves the objective value. After examining all initial tuple matches, GREEDY finalizes the evidence mapping and creates the explanations in the same way as RSWOOSH and THRESHOLD.

**EXACTCOVER:** We create a final baseline by adapting the integer programming solution of the Exact Cover problem to solve the EXP-3D problem as follows: we map tuples in one provenance relation as elements, and tuples in the other provenance relation as sets; an element is covered by a set if there exists an initial tuple mapping between their corresponding tuples. We further adapt

<sup>13</sup>We have also conducted experiments using Jaro similarity, but its performance is strictly inferior to Jaccard similarity in all experiments, so we don’t report it in the graphs.

the objective function of the Exact Cover problem from a decision problem to an optimization problem, where we want to find a collection of sets such that the total number of covered sets and elements is maximized.

**EXPLAIN3D:** Our proposed system, explain3D, expresses and optimizes the problem as linear constraints and solves the constructed MILP problem(s) through a MILP solver (Section 3, Section 4).

### 5.1.4 Metrics

**Explanation accuracy:** We evaluate the explanation accuracy of the algorithms using precision, recall, and F-measure. We calculate precision as the fraction of true explanations over derived explanations, and recall as the fraction of true explanations over the gold standard; F-measure is the harmonic mean of precision and recall ( $\frac{2 * precision * recall}{precision + recall}$ ).

**Evidence accuracy:** We also evaluate the evidence mapping accuracy with the same metrics. Similarly, we calculate the precision as the fraction of true tuple matches over the refined tuple matches, and recall as the fraction of true matches over the gold standard; F-measure as the harmonic mean of precision and recall.

**Execution time:** We evaluate the efficiency of all alternative algorithms through their total execution times, including the time for generating initial tuple matches.

## 5.2 Real-world datasets

We evaluate all the algorithms (Section 5.1.3) on both the Academic and IMDb datasets. Figures 6a, 6d, and 7a demonstrate the

precision, recall, and F-measure of the derived explanations; Figures 6b, 6e, and 7b demonstrate the precision, recall, and F-measure of derived evidence mapping; Figures 6c, 6f, and 7c demonstrate the total execution time.

**Single-dataset explanations.** Our evaluation with FORMALEXP examines whether single-dataset explanation solutions could address explanations across different datasets. This method does not generate an evidence mapping, and the derived explanations focus on why a query result is high or low, rather than why it is higher or lower than the other corresponding query. The why-high/why-low explanation question is a best-effort adaptation of this solution to our problem setting, but it is not a good enough proxy of the correspondence information encoded in the queries. As a result, the f-measure of FORMALEXP-Top15 is low, indicating that it is ill-suited for this problem setting.

**Record-linkage approaches.** Record-linkage methods do not generate explanations as a goal, but the tuple mappings they produce can be used as an evidence mapping and then mapped to explanations. RSWOOSH and THRESHOLD-0.9 produce evidence mappings with very high precision because they employ thresholds in refining the mappings (thus maintaining the most likely ones). However, their recall is low because they eliminate correct mappings that happen to have low probabilities. As these techniques miss many correct mappings, they include a large number of tuples in the explanations, thus resulting in low explanation precision.

Since RSWOOSH and THRESHOLD-0.9 employ thresholds in refining the mappings, they perform better when the initial mappings are of better quality, as is the case for the IMDB datasets. Their performance drops significantly in the Academic datasets. Through manual analysis, we noted that the initial tuple mappings in the academic data misses or has low probabilities for a significant portion of true matches. For example, the true tuple mapping, (“Foodservice Systems Administration”, “Food Business Management”) is absent from the initial mapping. Such cases are common in the academic datasets, but uncommon in the IMDB data because movie titles, persons’ names, and other attributes are less ambiguous. Further, our view generation and error injection only contributed relatively small perturbations, making matches easier to identify with higher accuracy.

GREEDY is also a record linkage approach, but uses our objective function instead of a strict threshold; thus, it is able to identify a larger portion of true mappings and has a higher recall. However, it may easily reach a local maximum, which results in lower precision and recall on the evidence mapping and further hurts the explanation accuracy. GREEDY is also impacted by the initial mapping quality, but is a bit more robust to it compared to RSWOOSH.

Ultimately, record linkage methods also are oblivious to the correspondence implied by the input queries. Failing to leverage this information, their effectiveness remains relatively low (below 0.8 f-measure), even in the most favorable data settings.

**EXPLAIN3D.** Our experiments demonstrate that our framework is highly accurate, with respect to both explanations and evidence mappings. Its superior performance compared to the other two categories of approaches is due to two main reasons. First, its objective function is cognizant of the query associations, in that it does not only focus on maximizing the quality of the matched tuples, but also seeks to minimize the unmatched tuples. As a result, it produces smaller explanations and identifies more correct mappings. As an example of the distinction from record linkage, consider two datasets of two tuples:  $A, B$  and  $A', B'$ . Suppose that the initial probabilistic tuple mapping is  $\{(A, A', 0.8), (B, B', 0.8), (A, B', 0.9), \text{ and } (B, A', 0.5)\}$ . Typical record linkage methods would select  $(A, B')$  as the single match, because it maximizes the

probability of the matched tuples. In contrast, EXPLAIN3D will derive the correct true mappings,  $(A, A')$  and  $(B, B')$ , because it considers explanation optimality by avoiding un-matched tuples. Second, record linkage methods often consider unmatched values as a very negative signal for matching a pair of tuples. In contrast, EXPLAIN3D does not weigh these mismatches as negatively, as it considers them as possible value-based explanations. As a result, EXPLAIN3D is more robust to variations in the quality of the initial tuple mapping. Nevertheless, the quality of the initial mapping does play a role, thus EXPLAIN3D performs better on the IMDB data than the academic datasets. However, in all cases, its accuracy is superior to the other methods.

Finally, while Exact Cover relates to EXP-3D through the NP-completeness reduction, it performs badly in all settings. This is expected since the Exact Cover problem does not consider tuple impacts, and does not refine the quality of the initial tuple mappings.

**Efficiency.** We show the total execution time of all methods in Figures 6c, 6f, and 7c. All methods are very efficient, with under a second runtimes. THRESHOLD, GREEDY, RSWOOSH, EXACTCOVER, and EXPLAIN3D rely on the same procedure to derive the input tuple matches, which takes more than 98% of their total execution time. EXACTCOVER scales better than the unoptimized version of EXPLAIN3D, because it has simpler problem settings. Figure 7c also demonstrates the effect of partitioning on the IMDB data. Partitioning allows EXPLAIN3D to scale effectively, without impact on its accuracy (Figures 7a and 7b).

### 5.3 Synthetic datasets

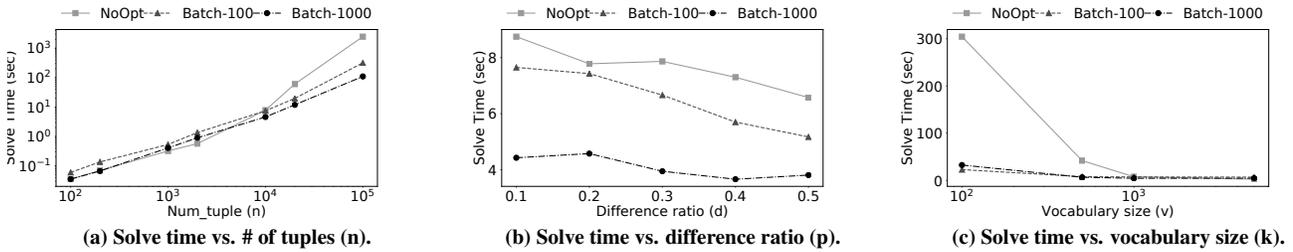
To stress-test EXPLAIN3D and evaluate its smart-partitioning optimization, we create a synthetic data generator to produce datasets and queries with diverse properties. In the synthetic data generator, we use the same schema and queries for every pair of datasets:

Dataset 1	Dataset 2
$Table(id, match\_attr, val)$	$Table(id, match\_attr, val)$
	$(match\_attr) \equiv (match\_attr)$
$Q_1 :$	$Q_2 :$
SELECT SUM(val) FROM Table;	SELECT SUM(val) FROM Table;

Based on the above schema, we follow three steps to produce a pair of datasets with the specified properties: (1) We first create  $n$  tuples with random attribute values and add them to both datasets. (2) We then randomly drop  $d$  percent of tuples, with uniform probability across tuples. (3) We randomly select  $d$  percent of tuples, again with uniform probability, and corrupt the tuples’ “val” attribute. To generate random values in the “match\_attr” attribute, we first create a vocabulary containing  $v > 5$  random words and then generate phrases, each of which consists of 5 random words from the vocabulary, as the attribute values; To generate random values in the “val” attribute, we randomly select an integer in the range of  $[1, 10]$ . The optimal explanations include tuples we dropped or corrupted in the steps (2) and (3); the optimal evidence can be easily derived from step (1). In this experiment, we study the performance of smart-partitioning by dynamically changing the number of partitions ( $k \in \mathbb{N}_{>1}$ , Definition 2) using a fixed batch size:  $k = \lceil \frac{|T_1| + |T_2|}{batch\_size} \rceil$ .

We evaluate EXPLAIN3D on three different settings: (1) the basic algorithm without the smart-partitioning optimization (NOOPT), (2) the optimized algorithm with batch size 100 (BATCH-100), and (3) the optimized algorithm with batch size 1000 (BATCH-1000). Figure 8 demonstrates the performance of NOOPT, BATCH-100, and BATCH-1000 over diverse parameter settings.

**Adjusting number of tuples ( $n$ ):** We first adjust the number of tuples ( $n$ ) in the synthetic datasets from 100 to 100K with fixed



**Figure 8: Efficiency performance of NOOPT, BATCH-100, and BATCH-1000 over synthetic datasets with diverse properties. Note that we only evaluate the solve time instead of the total execution time since the all methods share the same initial tuple matches generation time.**

difference ratio  $d = 0.2$  and vocabulary size  $v = 1K$ . As shown in Figure 8a, NOOPT performs well for problems with fewer tuples as the problem can be efficiently solved by a single MILP problem. However, its execution time grows quadratically, if not exponentially, with increasing data size. BATCH-100 and BATCH-1000 solve multiple MILP problems with bounded sizes, thus their solve time grows linearly with increasing number of tuples. Meanwhile, BATCH-1000 is significantly more efficient than BATCH-100 as BATCH-100 requires longer time to initialize and solve each individual sub-problems. With the smart-partitioning optimization, BATCH-1000 is more than  $20\times$  faster than NOOPT on problems with  $100K$  tuples.

**Adjusting difference ratio ( $d$ ):** We next adjust the difference ratio ( $d$ ) from 0.1 to 0.5 while keeping the other parameters fixed:  $n = 1K, v = 1K$ . As expected, all three methods require longer time for problems with lower difference ratio. This is because with higher difference ratio, there will be fewer tuples remaining in the datasets. Again, BATCH-1000 is much more efficient than BATCH-100 and NOOPT.

**Adjusting vocabulary size ( $v$ ):** Finally, we adjust the vocabulary size ( $v$ ) from 100 to  $10K$  and keep  $n = 1K, d = 0.2$ . In the synthetic data generator, we generate the value of the “match\_attr” attribute by randomly selecting 5 words from the vocabulary. Thus the probability that two tuples share at least one common word increases with lower vocabulary sizes. In other words, there will be many more initial tuple matches when we set  $v = 100$  than  $v = 10K$ . As shown in Figure 8c, BATCH-100 is  $15\times$  faster than NOOPT and even outperforms BATCH-1000 when  $v = 100$ . This is because the number of tuple matches in each sub-problem also affects the problem’s overall complexity. Thus, we need to divide the problem into smaller partitions when there is a larger number of initial tuple matches. With increasing vocabulary size (and decreasing number of tuple matches), BATCH-1000 starts to outperform the other two methods. When we increase the vocabulary size to a large enough number, e.g.,  $v = 10K$ , NOOPT, BATCH-1000, BATCH-100 start to perform similarly.

In all experiments on the synthetic datasets, NOOPT, BATCH-100, and BATCH-1000 achieve near perfect accuracy in the derived explanations and evidence mapping.

## 6. RELATED WORK

In this paper, we study the problem of explaining the disagreements in the results of semantically similar queries over disjoint datasets. While there is a growing body of work in data management research on deriving explanations, existing work focuses on one dataset at a time, and cannot address disagreements across datasets with potentially different schemas. Explain<sub>3D</sub> is, to the best of our knowledge, the first framework of its kind, that handles disagreements across disjoint datasets.

Data management research on explanations has focused on the assumption that data resides in a single dataset. The Scorpion system [56] finds predicates on the input data as explanations for a labeled set of outlier points in an aggregate query over a single relation. Roy and Suciu [45] extended explanations with a formal framework that handles complex SQL queries and database schemas involving multiple relations and functional dependencies. This explanation tool does not require any preparation for the data and derives the explanations as a set of conjunctive predicates. Roy, Orr and Suciu [44] further extend their work to provide richer and more insightful explanations on datasets with prepared candidate explanations derived by domain experts.

Other explanation work investigates the absence of answers from a query result [14, 47, 49]; these systems provide why-not explanations and sometimes modification suggestions to the queries. Work on provenance and causality [12, 25, 39] focuses on identifying the tuples that contribute to a query, and quantify their contributions. Finally, application-specific explanations focus on a particular domain, such as performance of MapReduce jobs [35], item rating [15, 48], and auditing and security [4, 22].

To compare two semantically similar queries and the corresponding databases, explain<sub>3D</sub> leverages existing schema matching techniques [7, 18, 28, 38] to derive the correspondence among attributes in two semantically correlated schemas. Existing schema matching solutions leverage a wide variety of techniques, from heuristics [18], to rules [38], to learning-based approaches [7, 28].

Another essential input for explain<sub>3D</sub> is the initial tuple matches (or the tuple mapping). We may acquire such initial tuple matches by leveraging existing entity resolution (or record linkage) techniques [5, 8, 16, 20, 54]. More specifically, explain<sub>3D</sub> treats existing entity resolution approaches as blackboxes and uses them to derive the matches and include them as part of the input.

## 7. SUMMARY OF CONTRIBUTIONS

In this paper, we presented an effective and scalable framework, explain<sub>3D</sub>, that derives explanations for the disagreements between the results of two semantically similar queries over two disjoint datasets. Our work formalized several important concepts and essential properties that explanations should satisfy. Explain<sub>3D</sub> uses a novel formalization and models explanations as two generic types, provenance-based explanations and value-based explanations, and evaluates the quality of explanations through a probabilistic model. The core stage of explain<sub>3D</sub> is a translation of the explanation problem into a mixed integer linear program, allowing the use of modern constrained solvers to address it. Our work further introduced a smart-partitioning optimization that allows explain<sub>3D</sub> to scale to large data sizes. To the best of our knowledge, explain<sub>3D</sub> is the first explanation framework that can address disagreeing query results across disjoint datasets.

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