Reasoning about Spatial Relationships in Picture Retrieval Systems

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Abstract

In this paper, we consider various spatial relationships that are of general interest in pictorial database systems. We present a set of rules that allow us to deduce new relationships from a given set of relationships. A deductive mechanism using these rules can be used in query processing systems that retrieve pictures by content. The given set of rules are shown to be *sound*, i.e. the deductions are logically correct. The rules are also shown to be *complete* for three dimensional systems, i.e. every relationship which is implied by a given consistent set of relationships F is deducible from F using the given rules. In addition, we show that the given set of rules is incomplete for two dimensional systems.

1 Introduction

We are currently witnessing an explosion of interest in multimedia technology. Consequently, pictorial and video databases will become central components of many future applications. Access to such databases will be facilitated by a query processing mechanism that retrieves pictures based on user queries. In this paper, for the first time, we present a deductive method to reason about spatial properties of objects in pictorial databases. A component based on this method can be used as part of a picture retrieval system.

We assume that there is a database containing the pictures. We also assume that each picture is associated with some meta-data describing the contents of the picture. This meta-data contains information about the objects in the picture, their properties and the relationships among them. For example, consider a picture containing a man shaking hands with a woman in front of a building. The meta-data about this picture identifies three objects, man, woman and the building, and the spatial relationship *in-front-of* and the non-spatial relationship hand-shaking. We assume that this meta-data is generated a priori (possibly, by image processing algorithms, or manually, or by a combination of both), and is stored in a separate database. This meta-data will be used by the query processing mechanism in determining the pictures that need to be retrieved in response to a query. The meta-data facilitates efficient query processing, i.e. it avoids the invocation of the expensive image processing algorithms each time a query is processed.

The meta-data stored in the database may not be complete in the following sense. For example, it may contain the relationships that object A is *behind* object B, and B is *behind* C, but not the relationship that A is *behind* C. Note that the last relationship is implied by the first two. This incompleteness may be due to any of the following reasons. Firstly, existing image processing algorithms may not be able to recognize all objects and their relationships. For example, since pictures are two dimensional images of three dimensional scenes, some of the spatial relationships in the missing dimension may not be detectable by the image

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processing algorithms. The missing objects and/or the missing relationships may have been introduced manually. In this process some of the implied relationships may have been left out to save time for entering these relationships manually. Secondly, the implied relationships may not have been stored explicitly in order to save space. This saving in space will be advantageous in distributed environments (for example, see [YGB89, LY93]) where the meta-data is stored at the user sites with limited storage facilities, while the actual pictures are stored in remote sites. The query processing mechanism executed at the user sites uses the meta-data to determine the pictures that need to be retrieved from the remote sites.

In this paper, we consider various spatial relationships that are of general interest in picture retrieval systems. Specifically, the following relationships— *left_of*, *right_of*, *in_front_of*, *behind*, *above*, *below*, *inside*, *outside* and *overlaps* — are investigated. We present a set of rules R that allow us to deduce new relationships from a given set of relationships. These rules are shown to be *sound*, i.e. the deductions are logically correct. Furthermore, the set of rules are shown to be *complete* for three dimensional systems, i.e. every relationship which is implied by a given consistent set of relationships F is deducible from F using the rules in R. In addition, we show that the set of rules R is incomplete for two dimensional systems.

Existing pictorial database management systems have been mostly application dependent (for example see [Amd93, LeeW93, RP92, CHH93]). Some preliminary work towards a unified framework for content based retrieval of images can be found in [GWJ91, GRV94]. Our motivation is to construct a general-purpose pictorial retrieval system which will accommodate pictorial databases for a broad *class* of applications. In order to accomplish this, we need to devise a powerful set of tools. These tools consist of components to reason about spatial relationships, to handle user interfaces, and to compute a degree of similarity between a query and a picture etc. In this paper, we concentrate on the reasoning about spatial relationships. Earlier work on handling spatial relationships are mostly based on algorithms [CSY84, CCT94, GR94]. In these approches, new algorithms need to be devised whenever additional spatial operators are added to the existing systems. On the other hand, our approach is to construct rules which allow spatial relationships to be deduced (The deduction mechanism itself can be carried out by existing systems such as LDL and CORAL [NT89, TZ86, R92]). If new spatial operators are employed, it is sufficient to augment existing rules with additional rules that capture the interactions between the new and the existing spatial operators. The rules for the existing operators need not be changed. Another difference between our approach and earlier approaches is that the sets of spatial operators are not identical. For example, we employ the operators *overlaps*, *inside*, *outside*, which are not present in the earlier approach. On the other hand, some measure of distance was provided, but is not considered in this paper. Our intention is to first provide a set of deductive rules for basic spatial operators. Then, the basic set of operators will be enlarged so as to be suitable for different applications. Surveys of pictorial database systems can be found in [TaY84, GrM92, ChH92].

In [MS93a, MS93b], Marcus and Subrahmanian present a general formal framework for multimedia systems. The model of pictorial databases that we use in this paper can be considered as a special case of media-instances defined in their work. For example, our pictures and objects in pictures correspond, respectively, with the states and features in their paper. While they provide a general framework for multimedia systems, we give a complete axiomatization for deducing spatial relationships of 3-dimensional pictures. There has also been much work done on defining and handling queries involving spatial relationships in spatial databases such as Geographic Information Systems (see [Eg89] for references). However, none of these works provides an axiomatization for deducing spatial relationships.

The paper is organized as follows. Section 2 presents the notation and various definitions used in the remainder of the paper. This section also introduces a property called *connectedness* which is assumed to be satisfied by all the objects in the pictures. Section 3 gives the system of rules for the different spatial operators. It also presents the soundness result, and the completeness result for 3-dimensional pictures. In addition, it shows the incompleteness of the rules for 2-dimensional pictures by providing a counter example. Section 4 contains a discussion showing how the completeness (or incompleteness) results are affected by removing the connectedness assumption. We also discuss the addition of new operators. This section also discusses future work.

2 Notation and Definitions

Objects and Pictures

We assume that each object has a unique name associated with it belonging to a finite set of names N. Each 3-dimensional picture specifies a set of points occupied by each object present in the picture. Formally, a 3-dimensional picture p is a partial function that maps each object in N to a non-empty set of points in the 3-dimensional Cartesian space; here each point in the 3-dimensional space is given by its three coordinates. For an object $A \in N$ and picture p, we say that A is present in the picture p iff $p(A)$ is defined. When the object A is present in the picture p, then we use the term "object A in the picture p" to refer to the set of points $p(A)$. A 2-dimensional picture is a partial function that maps each object to a set of points in the 2-dimensional space.

We say that an object A is *connected* in the picture p if A is present in the picture, and for every pair of points in $p(A)$ there exists a line joining the two points such that all the points on the line are also in $p(A)$; the line connecting the two points need not be a straight line [1]. The connectedness property prevents an object from having disjoint parts. We say that a picture is *connected* if every object defined in the picture is connected. If an object is connected in a picture then the set of x-coordinates of all the points in the object form a single interval on the x-axis; similar conditions hold for the set of y- and z-coordinates; these intervals can be open or closed on either side. We say that a connected object is *closed* if these x, y and z intervals are closed intervals. Unless it is otherwise stated, we assume that all the pictures that we consider are connected and all objects in them are closed. We will later discuss the consequences of removing this assumption.

Spatial Relationships

As indicated in the introduction, we consider the following set of spatial relationship symbols— *left_of, right_of, behind, in_front_of, above, below, inside, outside*, and *overlaps*.

Let p be a picture in which objects A and B are defined. Now, we formally define when p satisfies the above relationships. If x is any of the above relationship symbols, we write $p \models A\ x\ B$ to denote that p satisfies the relationship $A\ x\ B$.

- $p \models A\ left\_of\ B$, informally stated that A is to the left of B in the picture p, iff the x-coordinate of every point in $p(A)$ is less than the x-coordinate of every point in $p(B)$.
- $p \models A\ above\ B$, informally stated that A is above B in the picture p, iff the y-coordinate of every point in $p(A)$ is greater than the y-coordinate of every point in $p(B)$.
- $p \models A\ behind\ B$, informally stated that A is behind B in the picture p, iff the z-coordinate of every point in $p(A)$ is greater than the z-coordinate of every point in $p(B)$.
- $p \models A\ inside\ B$, informally stated that A is inside B in the picture p, iff $p(A) \subseteq p(B)$.
- $p \models A\ outside\ B$, informally stated that A is outside B in the picture p, iff $p(A) \cap p(B) = \emptyset$. This means that A and B do not have any common points.
- $p \models A\ overlaps\ B$, informally stated that A. overlaps B in the picture p, iff $p(A) \cap p(B) \neq \emptyset$. This means that A and B have at least one common point.

The semantics of operators *right_of, in_front_of* and *below* are defined similarly.

It is to be noted that if A and B are present in the picture p, then A is outside B iff A and B do not overlap in p. Also, the relationship symbols *right_of, in_front_of, below* are duals of *left_of, behind* and *above*, respectively. For example, A is to the left of B in a picture iff B is to the right of A.

We use the term "relationship symbol" to refer to any of the symbols given previously. We use the term "relationship" to denote a triple of the form $A\ x\ B$ where A and B are objects, and x is a relationship symbol. Sometimes, we use the term relationship to refer to the corresponding symbols. However, the intended meaning will be clear from the context.

Deductive Systems

Let F be a finite set of relationships. We say that F is *consistent* if there exists a picture that satisfies all the relationships in F. For example, the set $\{A\ left\_of\ B\ ,\ B\ left\_of\ C\}$ is consistent, while the set $\{A\ overlaps\ B\ ,\ A\ outside\ B\}$ is inconsistent. We say that a relationship r is *implied by* F, if every picture that satisfies all the relationships in F, also satisfies the relationship r. For example, the set of relationships $\{A\ left\_of\ B\ ,\ B\ left\_of\ C\}$ implies $A\ left\_of\ C$.

In this paper, we present various rules for deducing new relationships from a given set of relationships. Each rule will be written as $r\ ::\ r_1, r_2, ..., r_k$. In this rule r is called the head of the rule and the list $r_1, ..., r_k$ is called the body of the rule. We say that a relationship r is *deducible in one step* from a set of relationships F using a rule, if r is the head of the rule, and each relationship in the body of the rule is contained in F. Let R be a set of rules and F be a set of relationships. We say that a relationship r is *deducible* from F using the rules in R, if r is in F, or there exists a finite sequence of relationships $r_1, ..., r_k$ ending with r, i.e. $r_k = r$, such that r_1 is deducible in one step from F using one of the rules in R, and for each $i = 2, ..., k$, r_i is deducible in one step from $F \cup \{r_1, ..., r_{i-1}\}$ using one of the rules in R.

Now, we define *soundness* and *completeness* of a set of rules R. A single rule in R is said to be *sound* if, every picture that satisfies all the relationships in the body of the rule also satisfies the relationship given by

[1] Note that the connectedness requirement is different from convexity since the later property requires that, for any two points with in an object, there be a straightline connecting the two points which is contained entirely with in the object.

the head of the rule. The set of rules R is sound if every rule in R is sound. We say that the set of rules R is *complete* if it satisfies the following property for every consistent set F of relationships: every relationship implied by F is deducible from F using the rules in R.

3 Rules for Deducing Spatial Relationships

Now, we present a system of rules R for deducing new spatial relationships from existing ones. In these rules, we exclude the relationship symbols *right_of*, *in_front_of* and *below*. As indicated before, these relationship symbols are duals of *left_of*, *behind* and *above*, respectively. They can be handled by simply introducing additional rules that relate them to their duals as indicated at the end of the section.

I. (Transitivity of *left_of*, *above*, *behind*, and *inside*): This rule indicates the transitivity of some of the relationships. For example, this rule allows one to deduce the relationship A *left_of* C from the relationships A *left_of* B and B *left_of* C. Let x denote any relationship symbol in {*left_of*, *above*, *behind*, *inside*}. We have the following rule for each such x.
$A\ x\ C$ :: $A\ x\ B$, $B\ x\ C$

II. This rule captures the interaction between the relationships involving *left_of*, *above*, *behind*, and the relationship involving *overlaps*. For example, it allows us to deduce A *left_of* D from the relationships A *left_of* B, B *overlaps* C and C *left_of* D. Let x denote any of the relationship symbols— *left_of*, *above* and *behind*. We have the following rule for each such x.
$A\ x\ D$ :: $A\ x\ B$, B *overlaps* C, $C\ x\ D$

III. This rule captures the interaction between the relationships involving *left_of*, *above*, *behind*, *outside*, and the relationship involving *inside*. For example, it allows one to deduce A *left_of* C from the relationships A *inside* B and B *left_of* C. The relationship A *left_of* C can also be deduced from A *left_of* B and C *inside* B. These two types of deductions are captured by rules (a) and (b) given below. Let x denote any relationship symbol in { *left_of*, *above*, *behind*, *outside*}. We have the following two rules for each such x.
(a) $A\ x\ C$ :: A *inside* B, $B\ x\ C$
(b) $A\ x\ C$ :: $A\ x\ B$, C *inside* B
Rule (b) is redundant for the case when x is the relationship symbol *outside*; for the other cases (a) and (b) are independent.

IV. (Symmetry of *overlaps* and *outside*): This rule captures the symmetry of *overlaps* and *outside*. Let x denote either of *overlaps* and *outside*. We have the following rule for each such x.
$A\ x\ B$ :: $B\ x\ A$

V. This rule allows one to deduce that two objects are outside each other if one of them is to the left of, or above, or behind the other object. Let x denote any of the relationship symbols in {*left_of*, *above*, *behind*}. We have the following rule for each such x.
A *outside* B :: $A\ x\ B$

VI. This rule allows one to deduce that if an object is inside another object, then the two objects overlap.
A *overlaps* B :: A *inside* B

VII. This rule allows one to deduce that A overlaps with B if B overlaps with an object inside A.
A *overlaps* B :: C *inside* A, C *overlaps* B

VIII. This rule says that every object is inside itself. Note that this rule has no body.
A *inside* A ::

Now, we prove the soundness of the rule system R given above for 2-dimensional and 3-dimensional pictures. We write $F \vdash r$ to indicate that the relationship r is deducible from F using the set of rules in R. For 2-dimensional pictures, we will not have the relationship *behind* and the rules referring to it.

THEOREM 1:(Soundness-theorem) The set of rules given by R is sound for 2 and 3-dimensional pictures.

Proof: Clearly, the rules given by I are sound. For example, assume that the relationships A *left_of* B, B *left_of* C are satisfied in a picture p, and let u, v and w be arbitrary points in $p(A), p(B)$ and $p(C)$ respectively. Clearly, x-coordinate of $u <$ x-coordinate of $v <$ x-coordinate of w. From transitivity of $<$, we see that the x-coordinate of $u <$ x-coordinate of w. Since u,v and w are arbitrary points, it follows that p satisfies the relationship A *left_of* C. The soundness of the rules given by IV is easy to see. To see the soundness of IIIa, assume that p is a picture satisfying the relationships A *inside* B and B *outside* C. Clearly, $p(A) \subseteq p(B)$ and $p(B) \cap p(C) = \emptyset$. Hence $p(A) \cap p(C) = \emptyset$ and p satisfies the relationship A *outside* C. Similar proofs can be given for other relationships such as *left_of*, *above* and *behind* and also for the different cases of IIIb.

To prove the soundness of the rules given by II, assume that p is a picture that satisfies the relationships A *left_of* B, B *overlaps* C and C *left_of* D. Let u and v be arbitrary points in $p(A)$ and $p(D)$, respectively. Also, let w be a point in $p(B) \cap p(C)$; there exists such a point w since $p(B) \cap p(C) \neq \emptyset$. Clearly, the x-coordinate of u is $<$ the x-coordinate of w which is $<$ the x-coordinate of v. Using the transitivity of $<$, it follows that the x-coordinate of u is $<$ x-coordinate of v. Since u and v are arbitrary points in $p(A)$ and $p(D)$ respectively, it follows that the p satisfies the relationship A *left_of* D.

It should be easy to see the soundness of the V. Suppose, the relationship A *left_of* B is satisfied in a picture p. Then, every point in $p(A)$ is strictly to the left of every point in $p(B)$, and hence $p(A) \cap p(B) = \emptyset$. As a consequence, A *outside* B is satisfied in p. The soundness of VI, VII and VIII should also be easy to see. □

Completeness for 3-dimensional Pictures

THEOREM 2:(Completeness-theorem) The set of rules R is complete for 3-dimensional pictures.

To prove the completeness theorem, we need the following definitions and lemmas.

In our proofs we use constructions that extend objects in the x-,y- and z-directions. These extensions are carried out so that the extended objects continue to satisfy many of the previous *left_of*, *above* and *behind* relationships. If the extensions are within certain bounds, then the previous relationships continue to be satisfied. With this as the main motive, for each picture p and object A, we define three open intervals *x_bounds*(A,p), *y_bounds*(A,p) and *z_bounds*(A,p) as follows.

We define the interval *x_bounds*(A,p) as follows. Let u and v respectively be the minimum and maximum x-coordinate of any point in $p(A)$. Let u' be the largest x-coordinate of any point in any object to the left of A in p, and v' be the smallest x-coordinate of any point in any object to the right of A in p. If there is no object to the left of A then take u' to be $-\infty$. Similarly, if there is no object to the right of A take v' to be ∞. Let u'' be the mid point between u and u', i.e. $u'' = (u+u')/2$. Let v'' be the mid point between v and v', i.e. $v'' = (v+v')/2$. Now define *x_bounds*(A,p) to be the open interval (u'', v'') on the real line. The other bounds *y_bounds*(A,p) and *z_bounds*(A,p) are defined using the y and z coordinates respectively. It should be easy to see that each of the above intervals contains an infinite number of points. Now, define *bounds*(A,p) to be the set of all points in the 3-dimensional space such that the x-coordinates, y-coordinates and z-coordinates of these points are contained within the intervals *x_bounds*(A,p), *y_bounds*(A,p) and *z_bounds*(A,p), respectively. Note that *bounds*(A,p) denotes an open rectangular box which is non-empty, and has non-zero volume. It should also be easy to see that $p(A) \subseteq$ *bounds*(A,p).

In any picture p, if object A is to the left of object B then all the values in *x_bounds*(A,p) are less than all the values in *x_bounds*(B,p). The following lemma is a consequence of the above observation. It states that if objects in the picture are changed so that the resulting objects are within their old bounds, then all of the previous *left_of*, *above*, *behind* relationships continue to be satisfied.

LEMMA 3: Let p and p' be two pictures containing the same objects. If for every object A, the x-coordinates of all points in $p'(A)$ are in *x_bounds*(A,p), then every *left_of* relationship satisfied in p is also satisfied in p'; similar conditions hold for *above* and *behind* relationships using *y_bounds*(A,p) and *z_bounds*(A,p) respectively. Also, if for every object A, $p'(A)$ is contained in *bounds*(A,p), then p' satisfies the same *left_of*, *above* and *behind* relationships as p.

We say that an object in a picture is a *line object* if it is a connected set of lines. The following lemma says that if a picture satisfies a set of relationships, then there exists another picture satisfying the same set of relationships and in which all objects are line objects.

LEMMA 4: For every picture p, there exists another picture p' such that all the objects in p' are line objects, and such that p' satisfies all the relationships satisfied by p.

Proof: Let p be any picture. We obtain p' as follows. The objects in p' are defined inside out, i.e. we first define those objects that do not have any other objects in them. Let A be any such object. For each object B such that A overlaps B in p we take a single point in $p(A) \cap p(B)$. (If A does not overlap with any object, then we take an arbitrary point in $p(A)$ and consider that to be $p'(A)$). Then, we join all these points by lines that are contained entirely within $p(A)$. Furthermore, we make the connecting lines go around any other lines belonging to previously defined objects in p' so that no additional overlaps are caused. If A has some objects inside it, we first define $p'(B)$ for all objects B inside A, inductively; then we choose a finite number of points in $p(A) \cap p(C)$ for each C that overlaps with A, but is not inside A, in p. Then, we join all these points, and all points in $p'(B)$ for each B which is inside A in p, by lines that are contained entirely within $p(A)$ and such that no additional overlaps are created. We can thus construct $p'(A)$ such that A is connected in p', $p'(A) \subseteq p(A)$, and all overlap relationships in p are also satisfied in p'. It is easy to see that every *outside*, *inside*, *left_of*, *behind*, *above* relationships satisfied in p are also satisfied in p'. Clearly, all objects in p' are line objects. □

Proof of Theorem 2: To prove the completeness theorem, we have to show that for every finite set F of consistent relationships and for every relationship r, the following property holds:

(*) if F implies r, then $F \vdash r$ (i.e., r is deducible from F).

To show (*) for any F and r, it is enough if we show the following:
(**) if r is not deducible from F, then F does not imply r, i.e. there exists a 3-dimensional picture p that satisfies all the relationships in F but does not satisfy the relationship r.

Let F be any finite consistent set of relationships and r be any relationship that is not deducible from F using the rules in R. Now, we construct a picture that satisfies all relationships in F, but does not satisfy r. Since F is consistent, there exists a picture p that satisfies all the relationships in F. Furthermore, using lemma 4, we can assume that all objects in p are line objects. If p does not satisfy r, then p is the required picture. So, assume that p satisfies r. Now, we show how p can be modified so that the modified picture p' satisfies all the relationships in F, but does not satisfy r. The way we modify F depends on the relationship r. We need the following definitions in the remainder of the proof.

For any object A, define *inside*(A) to be the set of all B such that B *inside* A is deducible from F. Also, define *strict_inside*(A) to be the set of all B such that B *inside* A is deducible from F, but A *inside* B is not. It is easy to see that $A \in$ *inside*(A) (due to rule VIII), but $A \notin$ *strict_inside*(A). We also define a simple equivalence relation $\equiv$ among objects as follows: $A \equiv B$ iff A *inside* B and B *inside* A are deducible. Let *class*(A) denote the equivalence class containing A. All objectswq in *class*(A) need to be identical in any picture satisfying the relationships in F. In our modifications, whenever we modify an object A, we extend every object B such that $A \in$ *inside*(B); the extension to B is done so as to connect it to the modified A and to contain it. Also, when we modify or redefine A, we automatically assume that all objects in *class*(A) are changed identically.

We divide the remainder of the proof into the following cases. We consider the difficult cases in the beginning.

case 1: r is the relationship X *above* Y. In this case, we alter p substantially. The idea behind this alteration is to extend the object Y in p in the y-direction (i.e. parallel to the y-axis) so that the relation r is violated. When we extend Y, we also need to shift and extend some other objects so that the relationships in F continue to be satisfied. Now let u be the lowest point in $p(X)$ (i.e. having the smallest y-coordinate), and v be the highest point in $p(Y)$. Clearly, u is above v in p. Now, let d be some positive value greater than the difference between the y-coordinates of u and v. Now, we modify p as follows. We call this modification *Extend_up*(Y, d). We will be using similar modifications in other cases also.

To describe the modification *Extend_up*(Y, d), we define two sets of objects *above*(Y) and *overlaps_above*(Y). The set *above*(Y) contains exactly all the objects A present in p such that the relationship A *above* Y is deducible from F. The set *overlaps_above*(Y) contains exactly the objects B such that $B \notin$ *above*(Y), and for some object $A \in$ *above*(Y), the relationship B *overlaps* A is deducible from F.

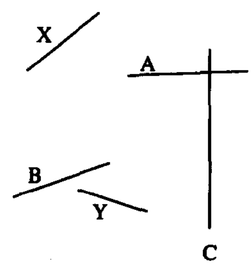
Now, the operation *Extend_up*(Y, d) modifies the picture p as follows. The only objects that will be altered are Y and those in *above*(Y) $\cup$ *overlaps_above*(Y). We extend Y from the point v by a straight line parallel to the y-axis so that the end point of this line is at a height d above v. Clearly, this end point is above the point u in $p(X)$. As a consequence X *above* Y is not satisfied in p'. The straight extension to object Y, as given above, may overlap with other objects in the picture and hence violate some *outside* relationships present in F. So, we modify this extension as follows. Since all objects in p are line objects, the obstacles to the above extension are going to be lines. Hence, we bend the straight extension to Y at the obstacles slightly so that it goes around these obstacles, and does not cause any undesired overlaps. Furthermore, we bend it at the obstacles slightly so that the x- and z-coordinates of all points in $p'(Y)$ are contained *x_bounds*(Y, p) and *z_bounds*(Y, p) respectively. This is illustrated in Figure 1.

Observe that we are able to do this, since *x_bounds*(Y, p) and *z_bounds*(Y, p) are intervals that are not single points.

We shift all objects in *above*(Y) in the y-direction by a constant distance $d' > d$ (see Figure 1), so that in the picture p', all the objects in *above*(Y) are going to be above all other objects. This shifting ensures that all the objects in *above*(Y) are above Y in p' also and such that they do not overlap some other objects, and hence violate some *outside* relationships. Note that all spatial relationships among the objects in *above*(Y) are preserved since they are all shifted by a constant distance in the y-direction.

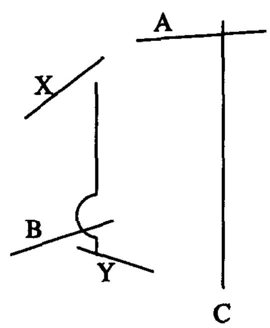
A in above(Y);

C in overlaps-above(Y).



before

Y is extended and bent to avoid overlapping B; X is not above Y; C is extended to continue overlapping A, A is shifted higher to remain above Y.



after

Figure 1 Illustrating proof of Theorem 2, case 1.

We extend each object $C \in$ *overlaps_above*(Y) as follows. For each object $A \in$ *above*(Y) such that the relationship C *overlaps* A is deducible from F, we extend C as follows. We take some point u where A and C overlap in p, i.e. some point $u \in p(A) \cap p(C)$, and extend C by a straight line parallel to the y-axis starting from u that goes around any obstacles so that the end point of this extended line is directly above u by a distance d'. Clearly, the end point of this extension will be in $p'(A)$ (see Figure 1). This ensures that C and A overlap in p'. Also, these extensions to C will be such that the x- and z-coordinates of points in $p'(C)$ are in *x_bounds*(C,p) and *z_bounds*(C,p) respectively. In addition, if $A \in$ *inside*(C) (i.e. A *inside* C is deducible from F), then we define $p'(C)$ so that $p'(C)$ contains $p(C)$ and the points in the previously defined extension and also all the points in $p'(A)$. This ensures that A *inside* C continues to be satisfied in p' also.

In the above construction, any extension done to any object D (such as Y or C as given in the previous two paragraphs) is also applied to all objects E such that D *inside* E is deducible from F. This ensures that all *inside* relationships deducible from F are preserved in p'.

Let p' be the picture obtained from p as specified above. Clearly, p' violates r. Now, we prove that p' satisfies all the relationships in F.

Claim: For every $r' \in F$, $p' \models r'$.

Proof of Claim: Consider any $r' \in F$. We prove that p' satisfies r' by cases.

r' is A *above* B: Note that for any object, the lowest point in p' is either same as in p, or it has been moved upwards. Hence, if B is not modified in p', i.e. same as in p, then A will continue to be above B in p' irrespective of whether A is modified in p' or not. Hence, if $p(B) = p'(B)$, then p' also satisfies r'. Now, assume that B is modified in p', i.e. $p'(B) \neq p(B)$. This can occur only if B is Y or $B \in$ *above*(Y) or $B \in$ *overlaps_above*(Y). If B is Y, then $A \in$ *above*(Y) and by construction of p', A would have been shifted up by a distance greater than the distance by which Y is extended, and hence p' will also satisfy r'. If $B \in$ *above*(Y) then $A \in$ *above*(Y) (due to rule I), and both A, B would have been shifted up by the same distance, and hence p' will also satisfy r'. If $B \in$ *overlaps_above*(Y) then, due to rule II in R, A will be in *above*(Y), and the distance by which B is extended up is exactly the distance by which A has been shifted, and hence p' will also satisfy r'.

r' is A *behind* B or A *left_of* B: In our construction we make sure that for every object C, the x- and z-coordinates of points in $p'(C)$ are within *x_bounds*(C,p) and *z_bounds*(C,p) respectively. From lemma 3, we see that p' satisfies r'.

r' is A *inside* B: If $B \in$ *above*(Y) then A is also in *above*(Y) by rule IIIa. In this case, both A and B are shifted by the same distance d', and hence r' continues to be satisfied in p'. If both A and B are not in *above*(Y), then any extension done to A is also applied to B, and hence r' continues to be satisfied. Now consider the subcase when $A \in$ *above*(Y) but $B \notin$ *above*(Y). Clearly, $B \in$ *overlaps_above*(Y), due to rule VI. Clearly, A is shifted in p' by a distance d'. By our construction, B is extended so that $p'(B)$ contains $p'(A)$ also. Hence r' continues to be satisfied in p'.

r' is A *overlaps* B: If both A and B are shifted (i.e. both of them are in *above*(Y)), then both of them would have been shifted by the same distance. If only one of them is shifted, then the other would have been extended by the same distance at one of the previous points of overlap. In all the above cases r' continues to hold. If neither of them is shifted, then whether they are extended or not, both of them contain their previous points of overlap, and hence p' satisfies r'.

r' is *A outside B*: If both A and B are shifted, then they are shifted by the same distance and hence they remain to be outside each other. If A is shifted and B is not shifted, then the distance by which A is shifted is long enough to not cause any overlaps with B; also, any extensions carried to B ensure that none of the outside relationships is violated. A symmetric argument takes care of the situation when B is shifted, but A is not. If neither of them is shifted, then any extensions carried out to them ensures that no overlap relation is violated. □

Case 2 of the Proof of theorem 2: r is X *behind* Y or X *left_of* Y. The proof is similar to case 1. If r is the relationship X *behind* Y, then we extend Y in the direction of the z-axis, shift some objects and extend some objects as in case 1. We call this operation $Extend\_back(Y, d)$ where d is appropriately chosen as in case 1. If r is the relationship X *left_of* Y, then we extend Y along the x-axis towards X, and also modify other objects using $Extend\_left(Y, d)$ operation where d is appropriately chosen.

Case 3: r is X *overlaps* Y. We will modify, i.e. redefine, some of the objects including X so that r is not satisfied. Recall that for any object A, *inside*(A) is the set of all objects B such that B *inside* A is deducible from F; *strict_inside*(A) is the set of all B such that B *inside* A is deducible and A *inside* B is not deducible from F. Also, for any object A, define *overlaps*(A) to be the set of all B such that B *overlaps* A is deducible from F.

We redefine all the objects $A \in$ *inside*(X). This redefinition starts with the innermost objects; we redefine them so that they do not overlap with Y, but continue to overlap with all other objects. Next we redefine each object $A \in$ *inside*(X) that contain the previously redefined objects; The redefined A is extended to join with, and also to contain, all objects that are supposed to be inside A. This process is carried out inductively until X itself is redefined.

Now, we redefine the objects in *inside*(X) as follows. First, consider any $A \in$ *inside*(X) such that *strict_inside*$(A) = \emptyset$. If A does not overlap with Y in p, then we do not change A, i.e. we let $p'(A) = p(A)$. Assume that A overlaps with Y in p. Now, we reconstruct A so that it continues to satisfy the required overlap relationships in F. For each object $B \in$ *overlaps*(A) such that $A \notin$ *strict_inside*(B), take any point x_B which is in $bounds(A,p) \cap bounds(B,p)$ but is not in $p(Y)$ [2] (Note that $B \notin$ *inside*(Y), otherwise due to the rule VII, A *overlaps* Y and hence X *overlaps* Y will be deducible from F which contradicts our assumption). If $x_B \notin p(B)$ then we extend B by a line to join with x_B and such that this line lies within $bounds(B,p)$ and such that no new overlaps are created (any object C such that B *inside* C is deducible from F is extended to contain the extended B). This gives us $p'(B)$. We obtain $p'(A)$ by taking each of the points x_B, as defined above, and join them with lines that lie within $bounds(A,p)$ and such that these lines do not overlap with any other objects.

Next, we define $p'(A)$ for each $A \in$ *inside*(X) such that $p'(B)$ has already been defined for all $B \in$ *strict_inside*(A). We construct $p'(A)$ as in the base case given above, and we further extend it to contain all points in $p'(B)$ for each $B \in$ *strict_inside*(A). We do this redefinition inductively until X itself is redefined. In this construction we ensure that $p'(A)$ is contained in $bounds(A,p)$ and no additional overlap conditions are satisfied.

Whenever we redefine or extend an object A, we need to extend all objects C such that $A \in$ *strict_inside*(C) but $C \notin$ *inside*(X) (i.e. A *inside* C is deducible from F but C *inside* X is not). Each such C is extended to join with A and to contain all the points in $p'(A)$.

Now, it should be obvious from the construction that r is not satisfied in p. We prove that every relationship in F is satisfied in p'. For every object A modified by the above construction it can be shown that $p'(A) \subseteq bounds(A,p)$. From this and lemma 3, we see that all the *left_of*, *above* and *behind* relationships satisfied in p are also satisfied in p'. We made sure that all *inside*, *outside* relationships deducible from F are preserved. Clearly, we preserved all *overlaps* relationships that are deducible from F.

Case 4: r is X *inside* Y. Let S be the set of all objects A such that X *inside* A is deducible from F using the rules in R. Clearly, $Y \notin S$ since we assumed that r is not deducible from F. Now, we extend X and all objects in S by a single small line all of whose lines within the bounds of the appropriate objects; this can be done since, all these objects are line objects. We can easily ensure that the extensions do not violate any *outside* relationships in F. It is easy to see that in the resulting picture all relationships in F are satisfied. Since the extended line is present in X but not in Y, it follows that the relationship r is not satisfied in the resulting picture.

[2]Such a point can be found because $bounds(A,p) \cap bounds(B,p)$ has non-zero volume, due to both of them being rectangular boxes open in all the directions such that they have non-empty intersection, while $p(Y)$ has zero volume because Y is a line object in p.

Case 5: r is X *outside* Y. Since r is not deducible from F, it follows that none of the relationships of the form $X\ x\ Y$ is deducible from F where $x \in \{left\_of,\ behind,\ above\}$ (If any of these relationships were deducible from F, then r itself will be deducible from F using rule V). Now, we extend X and Y so that all other (i.e. other than r) relationships between X and Y, and all their relationships with other objects, deducible from F, are preserved. The way the extensions are done depends upon how X and Y are positioned in p. The modifications to the picture are similar to those done in cases 1 and 2.

The following case is the most difficult case; other cases can be handled similarly. Assume that the relationships X *left_of* Y, X *above* Y and X *behind* Y are satisfied in p. Now take any point $u \in p(Y)$, and extend it horizontally towards X, by going around obstacles, such that the the x-coordinate of the end point of the extension is equal to the highest x-coordinate of any point in $p(X)$. As in cases 1 and 2, we use $Extend\_left(Y, d_x)$ operation where d_x is the difference between the x-coordinates of u, and the x-coordinate the rightmost point in $p(X)$. This shift preserves all relationships in F. Let u_1 denote the end point of the above extension, and p_1 be the resulting picture.

Now, extend Y from the point u_1 vertically upwards until the y-coordinate of the end point u_2 is equal to the y-coordinate of the lowest point in $p(X)$. Let d_y be the difference between the y-coordinate of the lowest point in $p(X)$ and the y-coordinate of u_1. To do this, we use the $Extend\_up(Y, d_y)$ operation as defined in case 1. This modification preserves all relationships in F. Let p_2 be the resulting picture.

Let d_z be the difference between the smallest z-coordinate of any point in $p(X)$ and the z-coordinate of u_2. Now extend Y from u_2 in the z-direction until the z-coordinate of the end point of the extension is equal to the smallest z-coordinate of any point in $p(X)$. This is achieved by the operation $Extend\_back(Y, d_z)$. Let u_3 be the end point of the above extension, and p_3 be the resulting picture. Now, it should be easy to see that we can take any point in $p_3(X)$ and join it with u_3 so that all the points on the line are in $bounds(p_3, X)$. It is not difficult to see that the above construction can be carried out such that the resulting picture p' satisfies all the relationships in F. Clearly, the point u_3 is going to be common to both the objects X and Y in p'. Hence the relationship r is not satisfied in p'. □

Addition of "right_of", "below" and "in_front_of"

Now, we show how to include the relationship symbols *right_of*, *below* and *in_front_of* into the deductive system. We simply add the following additional rules relating them to other relationship symbols.

IX. These rules say that *left_of* and *right_of* are duals.
A *right_of* B :: B *left_of* A
A *left_of* B :: B *right_of* A

X. These rules say that *above* and *below* are duals.
A *below* B :: B *above* A
A *above* B :: B *below* A

XI. These rules say that *behind* and *in_front_of* are duals.
A *behind* B :: B *in_front_of* A
A *in_front_of* B :: B *behind* A

The soundness of the above rules should be obvious. Let S denote all the rules from I through XI. Recall that R denotes the set of rules from I thorugh VIII. Now, we prove that the set of rules S is complete. Let F be any set of relationships involving all relationship symbols including *right_of*, *below* and *in_front_of*. Let r be any relationship implied by F, i.e. every picture p that satisfies F also satisfies r. Now we will show that r is deducible from F using the rules in S. Let E be the set of relationships obtained from F by replacing every relationship of the form A *right_of* B by B *left_of* A, every relationship of the form A *below* B by B *above* A and every relationship of the form A *in_front_of* B by B *behind* A. Now define a relationship r' which is a dual of of r as follows. If r is A *right_of* B, or is A *below* B, or is A *in_front_of* B then r' is B *left_of* A, or is B *above* A, or is B *behind* A, respectively. Otherwise, r' is r itself. From theorem 2, we see that r' is deducible from E using the rules in R. Also, all the relationships in E are deducible from F using the rules IX through XI. The relationship r' is also deducible from r using these rules. Putting all these observations together, we see that r is deducible from F using the rules in S.

Incompleteness for 2-dimensional pictures

In this section we show that the set of rules given by R is incomplete for 2-dimensional pictures. We give a counter example to show the incompleteness. First, using the relationship symbols that we have, we assert that an object X is enclosed between the objects A, B, C and D as shown in figure 2. To do this, we first state that the objects A, B overlap, B, C overlap, C, D overlap and D, A overlap. Next we state that A is to the left of X and X is to the left of C. We also state that B is above X and X is above D. All these relationships ensure that X is enclosed by A, B, C and D. Next we assert that another object Y overlaps with X, but is outside the objects A, B, C and D. This essentially means that Y also is enclosed by A, B, C and D. Then we assert that another object Z is to the left of A, B, C and D. Let F be the set of all these relationships. Clearly, Z is to the left of Y in this picture. It should be easy to see that every 2-dimensional picture, which is connected, and which satisfies all the relationships in F, also satisfies the relationship Z *left_of* Y, i.e. the relationship Z *left_of* Y is implied by the relationships in F. By exhaustively enumerating all the relationships that can be deduced from F using the rules in R, it is easy to see that Z *left_of* Y is not deducible from F using the rules given by R.

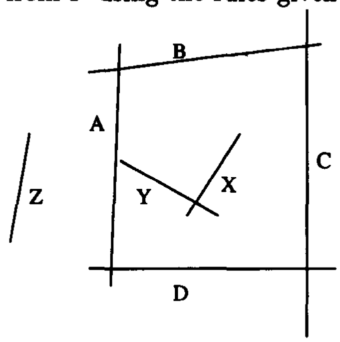


Figure 2 Illustrating incompleteness for 2-D objects; the objects are not necessarily lines.

4 Conclusions and Discussion

In this paper, for the first time, we have presented a deductive system for reasoning about a wide variety of spatial properties in picture retrieval systems. We have shown that our deductive system is complete for 3-dimensional pictures. We have also shown that this system is incomplete for 2-dimensional pictures.

We assumed that all objects in the pictures are connected and closed. The closedness assumption is made for technical convenience and is not crucial; the completeness continues to hold even if objects do not satisfy this property. On the other hand, the connectedness assumption is crucial. This property prevents an object from having disjoint parts. We briefly discuss the consequences of discarding the connectedness assumption. In this case, the deductive system presented in this paper can be shown to be complete for 3-dimensional as well as for 2-dimensional pictures. For 3-dimensional pictures, the completeness proof given in the paper continues to hold. (For 2-dimensional pictures, observe that the counter example given at the end section 3 and shown in figure 2, is no longer valid; that is, the set of relationships F given there no longer imply the relationship Z *outside* Y. To see this consider a picture in which Y has two disjoint parts, one part overlaps with X and is inside the area enclosed by A, B, C and D, the other part is outside this region and overlaps with Z.) Now we briefly show the construction of the completeness proof for 2-dimensional pictures when we discard the connectedness assumption. Consider the current proof for 3-dimensional pictures. In this proof, we assume that a relationship r is not deducible from a given set of relationships F, and construct a picture that satisfies all relationships in F but does not satisfy r. The only place where we use the 3-dimensional property is, when we make an extension/modification to an object to go around other objects (see figure 1), by using the third dimension, in order to avoid undesired overlap conditions. In such situations, since we no longer require the connectedness property, we simply make the object being extended to be discontinuous, i.e. one part of the object is on one side of the obstacle and the other part on the other side; then we do not need the third dimension to go around the obstacles. The closedness

Now, we would like to discuss other spatial operators. The operators *left_of, right_of, above, below, behind* and *in_front_of* considered in the paper are strict operators; that is, A *left_of* B is satisfied iff every point in A is strictly to the left of every point in B. Now, consider the addition of non-strict operators to the previous set of operators. The rules I,II,III,V,IX,X and XI in our deductive system (note that IX,X and XI are given at the end of section 3) involve the above mentioned strict operators. All these rules excepting V are sound for non-strict operators also. We extend these rules to include the non-strict operators. Rule V needs to be changed slightly for non-strict operators. After these modifications, by adding another set of rules, we can obtain a proof system for a set of operators consisting of the above strict operators, the corresponding non-strict operators, and the operators *inside, outside, overlaps*. This proof system can be shown to be complete for strictly 3-dimensional pictures (i.e. pictures in which all objects have non-zero volume).

In this paragraph, we briefly discuss how the spatial relationships and feature indices (see [MS93a]) can be used to retrieve pictures. We assign a similarity measure with each picture denoting how closely the picture matches the user description. The similarity measure is based on the objects and the relationships in common between the picture and the user description. We distinguish "fundamental" and implied relationships, assign different weights to them. To efficiently compute the similarity measures, we employ indices on the spatial relationships to retrieve ids of pictures. The details of this method, which uses the deductive system presented in this paper, will be addressed in a subsequent paper.

As part of the future work, it will be interesting to extend our rules so as to make the extended rules to be complete for 2-dimensional pictures that satisfy the connectedness property. Also, other application dependent spatial operators and deductive systems for them need to be explored.

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