REFERENTIAL INTEGRITY REVISITED : AN OBJECT–ORIENTED PERSPECTIVE *

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ABSTRACT

Referential integrity underlies the relational representation of object-oriented structures. The concept of referential integrity in relational databases is hindered by the confusion surrounding both the concept itself and its implementation by relational database management systems (RDBMS). Most of this confusion is caused by the diversity of relational representations for object-oriented structures. We examine the relationship between these representations and the structure of referential integrity constraints, and show that the controversial structures either do not occur or can be avoided in the relational representations of object-oriented structures.

Referential integrity is not supported uniformly by RDBMS products. Thus, referential integrity constraints can be specified in some RDBMSs non-procedurally (declaratively) , while in other RDBMSs they must be specified procedurally. Moreover, some RDBMSs do not allow the specification of certain referential integrity constraints. We discuss the referential integrity capabilities provided by three representative RDBMSs, DB2, SYBASE, and INGRES.

I. INTRODUCTION

The database design process involves specifying the objects and object connections relevant to the database application. In relational databases objects are represented by relation tuples, while object connections are represented by references between tuples. Such references are enforced in relational databases by ***referential integrity constraints*** [2]. There are two approaches to the specification of these constraints in relational databases. In the *Universal Relation* (UR) approach [11], referential integrity constraints are implied by associating different relations with common attributes; the referential integrity meaning of relations sharing common attributes is defined by a set of rules, called *UR assumptions*. The UR assumptions make the description of object structures extremely difficult and not entirely reliable, mainly because they require excessively complex attribute name assignments.

A different approach to the specification of referential integrity constraints is to associate explicitly a ***foreign-key*** in one relation with the ***primary-key*** of another relation [5]. Such constraints are a special case of ***inclusion dependencies*** [4]. Every explicit referential integrity constraint is usually associated with a ***referential integrity rule*** which defines the behavior of the relations involved in the constraint under insertion, deletion, and update. Explicit referential integrity constraints are easier to specify and understand than the implicit referential integrity constraints of the UR approach, because they are closer to the way users describe object structures. However, the referential integrity concept is still surrounded by confusion, as illustrated by the successive modifications of the original definition of [2] (e.g. see [3], [5]). Thus, certain referential integrity structures have unclear semantics, and therefore must be 'treated with caution' [6]. Obviously, a non technical user cannot be expected to manage the complexities of such a task.

In this paper we examine the characteristics of referential integrity constraints involved in the relational representation of object-oriented structures. We show that the controversial structures discussed in [6] can be avoided without any effect on the capability of relational schemas to represent object structures. We explore the characteristics of referential

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integrity constraints in the context of relational schemas representing *Extended Entity-Relationship* (EER) object structures. We have selected the EER model because of its widespread use in designing relational databases [19]. However, our results apply to any object-oriented data model that supports generalization and aggregation [9].

We have shown in [15] that an EER schema can be represented by a *Boyce-Codd Normal Form* (BCNF) relational schema of the form $(R, F \cup I)$, where R denotes a set of relation-schemes, and F and I denote sets of key and inclusion dependencies, respectively. Informally, relation-schemes represent object-sets, and inclusion dependencies represent the existence dependencies inherent to object connections. The inclusion dependencies in these schemas are *key–based*, that is, are referential integrity constraints, and relation-schemes correspond to either unique or multiple (embedded) object-sets. In [12] we have shown that the mapping process involved in representing EER schemas by relational schemas, can be expressed as the composition of (i) the mapping of EER schemas into a relational schemas, where every relation-scheme corresponds to a unique EER object-set, followed by (ii) relation-scheme mergings, that result in relation-schemes representing multiple object-sets.

We examine the structure of referential integrity constraints involved in relational schemas whose relation-schemes represent unique EER object-sets. We show that in such schemas referential integrity constraints can be associated with one out of four possible referential integrity rules. Next, we examine the effect of merging on the structure of referential integrity constraints, and show that merging entails associating some referential integrity constraints with an additional, fifth, referential integrity rule. In contrast, seven referential integrity rules are defined in [3] and [5]; we show that the two extra rules are not needed for representing EER object structures.

Currently, several relational database management systems (RDBMS), notably IBM's DB2, SYBASE, and INGRES, provide support for referential integrity. Interestingly, these systems provide different capabilities for specifying referential integrity constraints. Thus, DB2 [7] allows non-procedural (declarative) specifications of referential integrity constraints, but with certain restrictions on the allowed structures. Conversely, SYBASE [18] and INGRES [8] do not support declarative specifications of referential integrity constraints, and provide instead mechanisms (*triggers* in SYBASE and *rules* in INGRES) for specifying such constraints procedurally. We discuss in this paper the referential integrity capabilities of DB2, SYBASE, and INGRES. We show that some of the restrictions imposed by DB2 are too stringent. We compare the SYBASE and INGRES mechanisms for specifying referential integrity constraints, and discuss their limitations.

The paper is organized as follows. In section 2 we briefly review the relational and EER concepts used in this paper, and the relational representation of EER schemas. In section 3 we examine two controversial foreign-key structures in the context of relational schemas representing EER object structures. The semantics of referential integrity rules in the context of relational schemas representing EER schemas, is explored in section 4. In section 5 we examine the effect of merging relations on the structure of referential integrity constraints. The referential integrity capabilities of DB2, SYBASE, and INGRES are examined in section 6. We conclude with a summary. The procedures for mapping EER schemas into relational schemas, and for merging relation-schemes in relational schemas are given in the appendix.

II. Preliminary Definitions

In this section we review briefly the relational and Extended Entity-Relationship (EER) concepts used in this paper, and the representation of EER object structures using relational constructs.

2.1 Relational Concepts.

We use letters from the beginning of the alphabet to denote attributes and letters from the end of the alphabet to denote sets of attributes. We denote by t a tuple and by $t[W]$ the sub-tuple of t corresponding to the attributes of W.

A *relational schema* is a pair (R,Δ), where R is a set of relation-schemes and Δ is a set of dependencies over R. We consider relational schemas with $\Delta = F \cup I$, where F and I denote sets of functional and inclusion dependencies, respectively. A *relation-scheme* is a named set of attributes, $R_i(X_i)$, where R_i is the relation-scheme name and X_i denotes the set of attributes. Every attribute is assigned a *domain*, and every relation-scheme, $R_i(X_i)$, is assigned a *relation* (value), r_i. The *projection* of such a relation, r_i, on a subset of X_i, W, is denoted $\pi_W(r_i)$, and is equal to $\{t[W] \mid t \in r_i\}$. Two attributes are said to be *compatible* if they are associated with the same domain, and attribute sets X and Y are said to be *compatible* iff there exists a one-to-one correspondence of compatible attributes between X and Y.

Let $R_i(X_i)$ be a relation-scheme associated with relation r_i. A *functional dependency* over R_i is a statement of the form R_i: $Y \rightarrow Z$, where Y and Z are subsets of X_i; R_i: $Y \rightarrow Z$ is ***satisfied*** by r_i iff for any two tuples of r_i, t and t', $t[Y] = t'[Y]$ implies $t[Z] = t'[Z]$. A ***key*** associated with R_i is a subset of X_i, K_i, such that $R_i : K_i \rightarrow X_i$ is satisfied by any r_i associated with R_i and there does not exist any proper subset of K_i having this property. A relation-scheme can be associated with several ***candidate keys*** from which one ***primary-key*** is chosen.

Let $R_i(X_i)$ and $R_j(X_j)$ be two relation-schemes associated with relations r_i and r_j, respectively. An ***inclusion dependency*** is a statement of the form $R_i[Y] \subseteq R_j[Z]$, where Y and Z are compatible subsets of X_i and X_j, respectively; $R_i[Y] \subseteq R_j[Z]$ is ***satisfied*** by r_i and r_j iff $\pi_Y(r_i) \subseteq \pi_Z(r_j)$. If Z is the primary-key of R_j then $R_i[Y] \subseteq R_j[Z]$ is said to be ***key-based***, and Y is called a ***foreign-key*** of R_i. Key-based inclusion dependencies are ***referential integrity*** constraints ([2], [5]).

2.2 The Extended Entity-Relationship Model.

The basic concepts of the *Entity-Relationship* model, (***entity***, ***relationship***, ***attribute***, ***entity–set***, ***relationship–set***, ***value–set***, ***entity–identifier***, ***weak*** entity-set, relationship ***cardinality***, ***role***) have been repeatedly reviewed (e.g. [19]) since their original definition in [1]. We refer commonly to entities and relationships as ***objects***. The *Extended Entity-Relationship* (EER) model considered in this paper has two additional abstraction mechanisms, generalization and full aggregation. *Generalization* ([9], [19]) is an abstraction mechanism that allows viewing a set of entity-sets as a single ***generic*** entity-set. The inverse of generalization is called ***specialization***. The ***full*** capability of ***aggregation*** is provided in the EER model by allowing relationship-sets to associate any object-set, rather than only entity-sets.

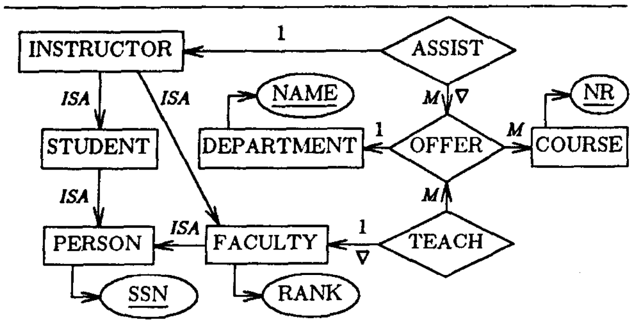


Fig.1 An Extended Entity-Relationship Schema.

An EER schema can be represented as an ***acyclic directed graph***, called *EER diagram*: entity-sets, relationship-sets, and attributes, are represented by rectangle, diamond, and ellipse shaped vertices, respectively; and the connection of EER object-sets and attributes is represented by directed edges. Thus, there are directed edges: from relationship-sets to the object-sets they associate, labeled by a cardinality (1 (***one***) or M (***many***)); from weak entity-sets to the entity-sets on which they depend for identification, labeled *ID*; from specialization entity-sets to generic entity-sets, labeled *ISA*; and from object-sets to their attributes. The acyclicity of EER diagrams is implied by certain restrictions satisfied by EER schemas ([9], [15]). A self-explanatory EER diagram example is shown in figure 1.

2.3 Relational Representation of Extended Entity–Relationship Schemas.

Relational schemas representing EER object structures are of the form $(R, F \cup I)$, where R, F, and I denote sets of relation-schemes, functional dependencies, and inclusion dependencies, respectively [15]. Informally, relation-schemes represent EER object-sets, inclusion dependencies represent the existence dependencies inherent to object-set connections, and functional dependencies represent entity-identifiers and relationship cardinalities. In [15] we have shown that an EER schema can be represented by a BCNF relational schema, such that every relation-scheme corresponds to a unique object-set, functional dependencies are ***key dependencies***, and inclusion dependencies are key-based, that is, are referential integrity constraints.

Relation–Scheme	Primary Key	Object–Set	EER Attribute : Attribute
$R_1(A_{1_1})$	A_{1_1}	PERSON	SSN : A_{1_1}
$R_2(A_{2_1})$	A_{2_1}	DEPARTMENT	NAME : A_{2_1}
$R_3(A_{3_1})$	A_{3_1}	COURSE	NR : A_{3_1}
$R_4(A_{4_1}, A_{4_2})$	A_{4_2}	FACULTY	RANK : A_{4_1}
$R_5(A_{5_1})$	A_{5_1}	STUDENT	
$R_6(A_{6_1})$	A_{6_1}	INSTRUCTOR	
$R_7(A_{7_1}, A_{7_2})$	A_{7_2}	OFFER	
$R_8(A_{8_1}, A_{8_2})$	A_{8_2}	TEACH	
$R_9(A_{9_1}, A_{9_2})$	A_{9_2}	ASSIST	

Referential Integrity Constraints

$R_4[A_{4_2}] \subseteq R_1[A_{1_1}]$	$R_5[A_{5_1}] \subseteq R_1[A_{1_1}]$
$R_6[A_{6_1}] \subseteq R_4[A_{4_2}]$	$R_6[A_{6_1}] \subseteq R_5[A_{5_1}]$
$R_7[A_{7_1}] \subseteq R_2[A_{2_1}]$	$R_7[A_{7_2}] \subseteq R_3[A_{3_1}]$
$R_8[A_{8_2}] \subseteq R_7[A_{7_2}]$	$R_8[A_{8_1}] \subseteq R_4[A_{4_2}]$
$R_9[A_{9_1}] \subseteq R_6[A_{6_1}]$	$R_9[A_{9_2}] \subseteq R_7[A_{7_2}]$

Fig.2 Relational Schema for EER Schema of Fig.1.

A procedure for mapping EER schemas into relational schemas based on the algorithms developed in [15], called ***Rmap***, is given in the appendix, and is exemplified in figure 2. Concerning the assignment of names to relational attributes, various techniques can be employed [16]. In order to keep ***Rmap*** independent of a specific name assignment for relational attributes, we use only symbolic names for attributes (e.g. A_{4_2} represents the second attribute of the fourth relation).

III. Partly Null and Overlapping Keys

In [6] Date discusses the problems caused by *overlapping*[†] (foreign and primary) keys, and *partly null*[‡] foreign-keys, and points out their obscure semantics. We show below that in relational schemas representing EER object structures overlapping keys do not occur, and partly null foreign-keys can be avoided.

3.1 Overlapping Keys.

As mentioned in the previous section, the relational schema representation of an EER object structure is of the form $(R, F \cup I)$, where R, F, and I denote sets of relation-schemes, functional dependencies, and inclusion dependencies, respectively. The following proposition shows that relational schemas generated by ***Rmap*** do not involve overlapping keys.

Proposition 1. Let $RS = (R, F \cup I)$ be a relational schema generated by ***Rmap***. Let $R_i(X_i)$ be a relation-scheme of R, and let FK_i denote the union of all foreign-keys, FK_{i_j}, associated with R_i. Then every foreign-key FK_{i_j} associated with R_i satisfies the following conditions: FK_{i_j} is either included in, or disjoint with, the primary-key of R_i; and FK_{i_j} is either equal to, or disjoint with, every other foreign-key of R_i.

Proof: see proposition 4.1 of [14]. ■

3.2 Partly Null Foreign–Keys.

We examine below the EER modeling of missing information, and then show how partly null foreign-keys can be avoided in relational schemas representing EER object structures.

The EER modeling of missing information involves allowing attributes to have *unknown* or *inapplicable* values, and allowing *partially specified* relationships, that is, allowing objects involved in relationships to be *unknown* or *inapplicable*. Consider the EER schema of figure 1, where entity-set FACULTY is associated with attribute RANK; the value of RANK can be *unknown* for some faculty members. If RANK, however, is associated with entity-set PERSON, then for a person who is not a faculty member, the RANK value is *inapplicable*.

Partially specified relationships are needed for representing embedded real-world associations. For example, suppose that a ternary relationship-set ASSIGNED involves entity-sets FACULTY, DEPARTMENT, and COURSE, and represents the assignment of faculty members to courses offered by departments; then in ASSIGNED relationships representing courses that are offered by departments, but that are not assigned to a faculty member, the FACULTY entity is *unknown*. Inapplicable attribute values and partially specified relationships can be avoided as follows:

- if an attribute A associated with entity-set E has *inapplicable* values for some entities of E, then A can be associated with a (possibly newly defined) specialization of E, E', so that A is always applicable for the entities of E'; for example, if attribute RANK above is associated with FACULTY (as shown in figure 1), then RANK is always applicable.
- if a relationship-set includes partially specified relationships, then it can be decomposed into independent or related (by aggregation) relationship-sets involving only fully specified relationships (note that binary relationship-sets consist only of fully specified relationships); for example, relationship-set ASSIGNED above can be decomposed into relationship-sets OFFER and TEACH (as shown in figure 1) that involve only fully specified relationships.

Consequently, only *unknown* attribute values are needed for the EER modeling of missing information, while partially specified relationships and *inapplicable* attribute values can be avoided. Note that for convenience entity-identifier attributes are usually not allowed to have *unknown* values.

Relational modeling of missing information employs special purpose *null* values. The various meanings associated with nulls are generally compressed into two [3]: *inapplicable* values and *unknown* (but applicable) values. From the discussion concerning the EER modeling of missing information, it follows that nulls representing *inapplicable* values can be avoided in databases associated with schemas representing EER object structures. Nulls representing *unknown* values, however, can represent

[†] Two sets of attributes, X and Y, are said to be overlapping iff $(X - Y)$, $(Y - X)$, and $(X \cap Y)$ are not empty.

[‡] A foreign-key of a relation-scheme R_i, Z, is said to be *partly null* if subtuples $t[Z]$ of relations associated with R_i, are allowed to contain both null and non-null values.

in such databases either an unknown EER attribute value or an unknown (relational) foreign-key attribute value. In relational schemas such as those generated by ***Rmap***, in which every relation-scheme corresponds to a unique object-set, primary-keys and foreign-keys are not allowed to have null values; these constraints represent the restrictions of not allowing object-identifiers to have unknown values, and not allowing relationships to be partially specified. Note that for foreign-keys these constraints are stronger than the *referential–integrity* constraint specified in [2]. However, in [3] Codd states that nulls should not be allowed for foreign-keys in most cases, but only when 'there exists a strong reason to depart from this discipline'. We show in section 5 that such an exception is needed only for relations that correspond to multiple, rather than single, object-sets.

IV. Referential Integrity Rules

In this section we discuss the existence dependencies inherent to object-oriented structures, and examine the referential integrity rules in the context of relational schemas representing such structures.

4.1 Existence Dependency Types.

Object-oriented structures imply certain existence dependencies that must be satisfied by updates. In an object-oriented environment an elementary update consists of modifying an attribute value, removing an object from an object-set, or adding a new object to an object-set. In order to satisfy the existence dependencies, usually an object x that is existence dependent on another object y, cannot be added before y is added, and y cannot be removed before x is removed. Intuitively, y *blocks* the addition of x, and x *blocks* the removal of y; therefore the existence dependency of x and y is said to be of type *block*. We propose an additional, type of existence dependency, called *trigger*: if an object x is *trigger* existence dependent on another object, y, then the removal of y *triggers* the removal of x (instead of x *blocking* the removal of y). Note that existence dependencies between objects do not affect attribute modifications, since such modifications are local to the objects.

The two types of existence dependency above are specified for pairs of object-sets, rather than pairs of individual objects, and can be represented in EER diagrams as follows: the *block* existence dependency is considered the default type, and therefore is not explicitly represented; the existence dependency of type *trigger* is represented by associating the edges connecting the corresponding object-sets with a '∇' label (see figure 1).

4.2 Referential Integrity Rules.

Let $R_i(X_i)$ and $R_j(X_j)$ be two relation-schemes associated with relations r_i and r_j, respectively. A referential integrity constraint $R_i[Y] \subseteq R_j[K_j]$ is associated with a ***referential integrity rule*** consisting of an ***insert–rule***, a ***delete–rule*** and an ***update–rule*** [5]. There is a unique insert-rule, ***restricted***, which asserts that inserting a tuple t into r_i can be performed only if the tuple of r_j referenced by t already exists. The delete and update rules define the effect of deleting (resp. updating the primary-key value in) a tuple t' of r_j : a ***restricted*** delete (resp. update) rule asserts that the deletion (resp. update) of t' cannot be performed if there exist tuples in r_i referencing t'; a ***cascades*** delete (resp. update) rule asserts that the deletion (resp. update) of t' implies deleting (resp. updating the subtuple $t[Y]$ in) the tuples of r_i referencing t'; and a ***nullifies*** delete (resp. update) rule asserts that the deletion (resp. update) of t' implies setting to null the subtuple $t[Y]$ in all the tuples t of r_i referencing t'.

Procedures mapping EER schemas into relational schemas, such as ***Rmap***, usually assume that existence dependencies are of type *block*. The semantics of such existence dependencies is expressed by associating the corresponding referential integrity constraints with *restricted* insert and delete rules. The new type of existence dependency introduced above, entails associating referential integrity constraints corresponding to *trigger* existence dependencies with *cascades* delete-rules. Finally, the preservation of existence dependencies under attribute modifications in object-oriented environments, is expressed by associating referential integrity constraints with *cascades* update-rules. Thus, the referential integrity constraints in figure 2, for example, must be associated with *restricted* insert-rules and *cascades* update-rules; the delete-rules are *restricted* for all the constraints with the exception of $R_8[A_{8_1}] \subseteq R_4[A_{4_2}]$ and $R_9[A_{9_2}] \subseteq R_7[A_{7_2}]$, for which the delete-rules are *cascades*.

The mapping of EER schemas into relational schemas can be straightforwardly extended with the explicit generation of referential integrity rules. Thus, procedure ***Rmap*** is extended by associating every referential integrity constraint *ref* with a *restricted* insert-rule and a *cascades* update-rule; the delete-rule depends on the type of existence dependency corresponding to *ref*: if the type of the existence dependency is *block* then *ref* is associated with a *restricted* delete-rule, otherwise (if the type is *trigger*) *ref* is associated with a *cascades* delete-rule.

4.3 Conflicting Existence Dependencies.

Objects can *block* or *trigger* the removal of other objects with which they are involved in existence dependencies not only directly, but also transitively. Thus, an object y can *trigger* the removal of an object x if y can *trigger* the removal of an object z on which x is *trigger* existence dependent; conversely, if x is *block* existent dependent on z then x *blocks* the removal of y. For example, suppose that in the EER schema of figure 1 the existence dependencies of INSTRUCTOR on STUDENT, STUDENT on PERSON, and FACULTY on PERSON, are of type *trigger*, while the existence dependency of INSTRUCTOR on FACULTY is of type *block*; then an INSTRUCTOR entity *blocks* the removal of a PERSON entity (via FACULTY), while a PERSON entity can *trigger* the removal of an INSTRUCTOR entity (via STUDENT).

If an object x *blocks* the removal of an object y which, in turn, can *trigger* the removal of x, then the result of removing y is unpredictable. In the example above, for instance, removing a PERSON entity depends on the order in which the removal is performed (i.e. first via FACULTY, or first via STUDENT). Consequently, objects should not be allowed to *block* the removal of objects that can *trigger* their removal. In terms of the EER object structure, this restriction can be stated as follows:

> Let O_i be an object-set of an EER schema ES, and let $Trig(O_i)$ be the set of object-sets of ES consisting of all the object-sets that are *trigger* existent dependent (directly or transitively) on O_i †. Then, for every O_i of ES, object-sets of $Trig(O_i)$ are not allowed to be (directly) *block* existent dependent either on O_i or on other object-sets of $Trig(O_i)$.

In the example above, for instance, *Trig*(PERSON) consists of FACULTY, STUDENT, and INSTRUCTOR, therefore the existence dependency of INSTRUCTOR on FACULTY cannot be of type *block*.

The following proposition defines the effect of the restriction above on the referential integrity rules involved in relational schemas representing EER object structures.

Proposition 2. Let $RS = (R, F \cup I)$ be a relational schema generated by ***Rmap***. Let $G_I = (V, H)$ be the referential integrity graph associated with RS, defined as follows: $V = R$, and $H = \{R_i \rightarrow R_j \mid R_i[Y] \subseteq R_j[Z] \in I\}$. Let $Casc(R_i)$ be the subset of R consisting of all the vertices that are connected to R_i in G_I by directed paths consisting of edges that correspond to referential integrity constraints associated with *cascades* delete-rules. Then for every R_i of R, the referential integrity constraints corresponding to the edges of G_I that connect vertices of $Casc(R_i)$ with R_i or other vertices of $Casc(R_i)$, are not allowed to be associated with *restricted* delete-rules.

Proof Sketch . ***Rmap*** generates referential integrity constraints associated with a referential integrity graph, G_I, that is isomorphic to a subgraph of the corresponding EER diagram (proposition 4.1 in [14]). The condition of the proposition follows from the restriction above specified for EER schemas. ■

† In EER diagram terms, every object-set of $Trig(O_i)$ is connected to O_i by a directed path consisting of ∇-labeled edges.

V. The Effect of Merging on Referential Integrity Constraints

In this section we examine the effect of merging relations on the structure of foreign-keys and referential integrity constraints.

Merging brings about the need to allow certain foreign-key attributes to have null values. We have shown in [12] that merging relations requires the introduction of additional ***null constraints*** [10] for restricting the way in which null values appear in merged relations. A procedure for merging relations in relational databases that preserves the information-capacity and the normal form of the corresponding schemas, has been proposed in [12]. The merging procedure developed in [12] may generate inclusion dependencies that are not key-based, that is, are not referential integrity constraints. We consider below a merging procedure that generates only key-based inclusion dependencies and involve only simple null constraints, that indicate the attributes that are not allowed to have null values.

A restricted version of the procedure proposed in [12] for merging relation-schemes in relational schemas, called ***Rmerge***, is given in the appendix. Given a schema $RS = (R, F \cup I)$ generated by ***Rmap***, and a subset $\bar{R}$ of R, such that the primary-keys associated with the relation-schemes of $\bar{R}$ are pairwise compatible, ***Rmerge*** maps RS into a new relational schema, $RS' = (R', F' \cup I')$, where R' results by replacing the relation-schemes of $\bar{R}$ with a new relation-scheme, R_m, and F' and I' consist of adjusted key and inclusion dependencies, respectively. Merging is achieved by outer-joining the relations associated with the relation-schemes of $\bar{R}$, so that the relation associated with R_m, r_m, includes tuples corresponding to every object represented in the merged relations. The dependencies associated with R_m ensure that the relations involved in merging can be reconstructed from r_m, so that the schema generated by ***Rmerge***, RS', has equivalent information-capacity with RS. An example of merging is shown in figure 3, where ***Rmerge*** is applied on the relational schema of figure 2 in order to merge relation-schemes R_7, R_8 and R_9 into R'_7.

Following merging, the referential integrity constraints involving the relation-schemes of $\overline{R}$ are replaced with referential integrity constraints involving the new relation-scheme, R_m, and some of the foreign-keys associated with R_m are allowed to have null values. Clearly, the referential integrity constraints involving R_m must be associated with *restricted* insert-rules and *cascades* update-rules, like the corresponding referential integrity constraints involving relation-schemes of $\overline{R}$. Concerning the delete-rules, ***Rmerge*** assumes that the referential integrity constraints involving relation-schemes of $\overline{R}$ are associated with *restricted* delete-rules, and therefore the referential integrity constraints involving R_m are also associated with *restricted* delete-rules. If *cascades* delete-rules are also considered, then ***Rmerge*** must be extended as follows:

- if a referential integrity constraint involving R_m, *ref*, is derived from a referential integrity constraint associated with a *cascades* delete-rule, then *ref* is associated either with (a) a *nullifies* delete-rule, if *ref* involves a foreign-key of R_m that is allowed to have null values, or (b) a *cascades* delete-rule, otherwise.

For example, referential integrity constraint $R'_7[A_{8_1}] \subseteq R_4[A_{4_2}]$ in the relational schema of figure 3, is associated with a *nullifies* delete-rule (because it corresponds to a referential integrity constraint associated with a *cascades* delete-rule and A_{8_1} is allowed to have null values), while the other referential integrity constraints involving relation-scheme R'_7 in this schema are associated with *restricted* delete-rules.

Relation-Scheme	Primary Key	Referential Integrity Constraints
$R_1(A_{1_1})$	A_{1_1}	$R_4[A_{4_2}] \subseteq R_1[A_{1_1}]$
$R_2(A_{2_1})$	A_{2_1}	$R_5[A_{5_1}] \subseteq R_1[A_{1_1}]$
$R_3(A_{3_1})$	A_{3_1}	$R_6[A_{6_1}] \subseteq R_4[A_{4_2}]$
$R_4(A_{4_1}, A_{4_2})$	A_{4_2}	$R_6[A_{6_1}] \subseteq R_5[A_{5_1}]$
$R_5(A_{5_1})$	A_{5_1}	$R'_7[A_{7_1}] \subseteq R_2[A_{2_1}]$
$R_6(A_{6_1})$	A_{6_1}	$R'_7[A_{7_2}] \subseteq R_3[A_{3_1}]$
$R'_7(A_{7_1}, A_{7_2}, A^*_{8_1}, A^*_{9_1})$	A_{7_2}	$R'_7[A_{8_1}] \subseteq R_4[A_{4_2}]$
		$R'_7[A_{9_1}] \subseteq R_6[A_{6_1}]$

Note : * nulls are allowed for A_{8_1} and A_{9_1}.

Fig. 3 Relational Schema of Fig. 2 after Merging.

rules.

The effect of merging on the structure of foreign-keys and referential integrity constraints is captured by the following proposition.

Proposition 3. Let $RS = (R, F \cup I)$ be a relational schema generated by ***Rmap***, and let $RS' = (R', F' \cup I')$ be the result of applying ***Rmerge*** on RS. Then (a) RS' satisfies the conditions of propositions 1 and 2. (b) In every relation-scheme $R'_i(X'_i)$ of R', if a foreign-key FK'_{i_j} is allowed to have null values then FK'_{i_j} consists of a single attribute that does not appear in any other foreign or primary key of R'_i. (c) If there exists a directed cycle in the referential integrity graph $G_{I'}$ associated with RS', then at least one of the referential integrity constraints corresponding to the edges of this cycle:
(i) involves a foreign-key that is allowed to have null values, and (ii) is associated with either a *restricted* or a *nullifies* delete-rule.

Proof Sketch. (a) The proof follows the specification of ***Rmerge***. (b) See proposition 4.2 in [14]. (c) ***Rmap*** generates referential integrity constraints associated with a referential integrity graph, G_I, that is isomorphic to a subgraph of the corresponding EER diagram. Since EER diagrams are acyclic (see [15]) G_I is also acyclic. Cycles in $G_{I'}$ may result from merging relation-schemes in RS, and the proof is based on the conditions of proposition 1 and the specification of ***Rmerge***. ■

The proposition above shows that merging does not alter the properties of propositions 1 and 2. Thus, relational schemas undergoing merging are still free of the undesirable foreign-key and referential integrity structures discussed in sections 3 and 4. It can be verified that proposition 3 is valid not only for the restricted merging procedure considered above, but also for extended merging procedures such as that defined in [12].

Several remarks concerning the referential integrity rules involved in the relational representation of object-oriented structures, are in order. Only *restricted* insert-rules, *restricted* delete-rules, and *cascades* update-rules are required for representing existence dependencies of type *block* between object-sets. *Cascades* delete-rules are required for representing existence dependencies of type *trigger*, and *nullifies* delete-rules are required only if relations are allowed to represent multiple object-sets (e.g. following merging). The referential integrity constraints involved in relational schemas representing object-oriented structures are always associated with *cascades* update-rules, therefore *nullifies* and *restrict* update-rules can be discarded. *Cascades* update-rules become superfluous if updates of primary-key attributes are not allowed. While such a restriction is unreasonable for regular relational attributes, this restriction underlies the definition of *surrogate* attributes [2]. Consequently, if surrogate attributes are used as primary and foreign key attributes, then *cascades* update-rules are not necessary.

VI. Referential Integrity in Relational Database Management Systems

Referential integrity is currently supported by several relational database management systems (RDBMS), notably IBM's DB2 [7], SYBASE [18], and INGRES [8]. The referential integrity capabilities of these three RDBMSs are briefly examined below. A more detailed analysis is provided in [13].

6.1 IBM's DB2.

In DB2 referential integrity constraints are specified non-procedurally (declaratively), but with certain restrictions. We examine below these restrictions and their effect on the relational representation of object-oriented structures.

Let $RS = (R, F \cup I)$ be a relational schema, where R, F, and I consist of relation-schemes, key dependencies, and referential integrity constraints, respectively; let G_I be the referential integrity graph associated with RS. The referential integrity constraints of I must satisfy the following restrictions in DB2:

1. Every referential integrity constraint of I is considered to be associated (by default) with a *restricted* update-rule.
2. Let I' be a subset of I that consists of referential integrity constraints corresponding to edges forming a directed cycle in G_I. If I' consists of a single constraint, then this constraint must be associated with a *cascades* delete-rule. If I' consists of two or more constraints, then at least two constraints of I' must be associated with *restricted* or *nullifies* delete-rules.
3. Let $Casc(R_i)$ be defined as in proposition 2 of section 4. For every pair of vertices of R, R_i and R_j, if R_j is connected in G_I by multiple edges to vertices of ($Casc(R_i) \cup \{R_i\}$), then the referential integrity constraints corresponding to these edges must be associated with identical, *restricted* or *cascades*, delete-rules. ■

Note that DB2 allows the specification of partly null foreign-keys and overlapping keys. We contrast below the DB2 restrictions above with the conditions satisfied by the referential integrity constraints of relational schemas representing EER object structures:

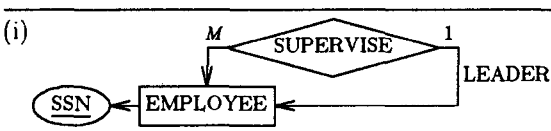
(i) DB2 does not support the *cascades* update-rules involved in the relational schemas representing EER object structures.

(ii) Restriction (2) above treats cycles consisting of single edges differently from cycles consisting of multiple edges. This apparent contradiction does not exist for the referential integrity constraints of relational schemas representing EER object structures (see condition (c) of proposition 3).

(iii) Restrictions (2) and (3) above are stronger than the conditions of propositions 1, 2, and 3. First, note that if every relation-scheme corresponds to a unique object-set, then restrictions (2) and (3) above are trivially satisfied. However, if relation-schemes are allowed to correspond to multiple object-sets (e.g. following mergings), then restrictions (2) and (3) above might not be satisfied. Consider, for example, the relational schemas of figures 4(ii) and 4(iv), generated by ***Rmap*** and ***Rmerge***, from the EER schemas of figures 4(i) and 4(iii), respectively. If the two existence dependencies of SUPERVISE on EMPLOYEE in the EER schema of figure 4(i) are both of type *block* (resp. *trigger*), then the referential integrity constraint in the relational schema of figure 4(ii) is associated with a *restricted* (resp. *nullifies*) delete-rule; consequently this schema does not satisfy restriction (2) above, while it satisfies the condition (c) of proposition 3.

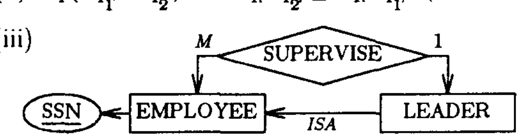
(iv) Conversely, the referential integrity constraints of the relational schema of figure 4(iv) satisfy

(i)



(ii) $R_1(A_{1_1}, A^*_{1_2})$, $R_1[A_{1_2}] \subseteq R_1[A_{1_1}]$ (Primary Key: A_{1_1})

(iii)



(iv) $R_1(A_{1_1}, A^*_{1_2})$, $R_1[A_{1_2}] \subseteq R_2[A_{2_1}]$ (Primary Key: A_{1_1})
$R_2(A_{2_1})$, $R_2[A_{2_1}] \subseteq R_1[A_{1_1}]$ (Primary Key: A_{2_1})

Notes: All relational attributes correspond to SSN.
* Nulls are allowed.

Fig. 4 Relational Schema Examples.

restriction (2) of the definition above, only if all the existence dependencies in the EER schema of figure 4(iii) are of type ***block***; however, if the existence dependencies are of type ***trigger***, then $R_1[A_{1_2}] \subseteq R_2[A_{2_1}]$ is associated with a ***nullifies*** delete-rule, while $R_2[A_{2_1}] \subseteq R_1[A_{1_1}]$ is associated with a ***cascades*** delete-rule; therefore, restriction (2) above is not satisfied, while condition (c) of proposition 3 is satisfied.

6.2 SYBASE and INGRES.

SYBASE [18] and INGRES [8] do not allow declarative specifications of referential integrity constraints. Instead, they provide mechanisms for specifying procedurally such constraints. We examine below the main characteristics and limitations of these mechanisms.

The mechanism provided by SYBASE for implementing referential integrity constraints involves ***triggers***. Triggers are a special kind of stored procedures that are activated (*fired*) when a relation is affected by a data manipulation (i.e., an insertion, deletion, or update). A trigger procedure is associated with a unique relation, say r_i, and employs two system provided relations, called ***deleted*** and ***inserted*** : the ***deleted*** relation consists of tuples of r_i that are going to be deleted or updated; the ***inserted*** relation consists of tuples that are going to be inserted into r_i, or newly updated tuples of r_i. SYBASE allows the specification of three trigger procedures per relation: an ***insert***, a ***delete***, and an ***update*** trigger procedure. Given relation r_i associated with relation-scheme R_i, the trigger procedures for r_i are executed in order to enforce the referential integrity constraints involving R_i. SYBASE imposes certain (rather arbitrary) technical limitations, such as the number of levels allowed for nesting triggers. Another limitation concerns the implementation of referential integrity constraints associated with ***cascades*** update-rules: a ***cascades*** update-rule can be implemented only if updates of primary-key attributes are restricted to single tuples at a time, that is, only if the ***inserted*** and ***deleted*** relations consist of single tuples.

The INGRES rule mechanism [8] is conceptually similar to the SYBASE trigger mechanism. The differences, which are mainly of a technical nature, are summarized below:

(i) Unlike SYBASE, INGRES does not restrict the number of triggers per relation, nor the depth of rule nesting.

(ii) While SYBASE triggers are set-oriented (i.e. are activated for sets of tuple manipulations), INGRES rules are tuple-oriented (i.e. are activated for single tuple manipulations); accordingly, INGRES rules present no problem in implementing cascades ***update−rules***.

(iii) INGRES provides a more evolved mechanism for handling errors and messages, and INGRES's ***Embedded SQL*** employed for specifying rules is more flexible than SYBASE's ***Transact−SQL*** employed for specifying triggers.

Overall, the INGRES rule mechanism is technically superior to the SYBASE trigger mechanism. However, both the SYBASE trigger and the INGRES rule mechanisms are extremely cumbersome. The use of SQL, compounded in SYBASE by certain syntactic limitations, make trigger and rule procedures very large and hard to comprehend. The manual specification of trigger and rule procedures is a tedious and error-prone process that tends to discourage users from specifying them for non-trivial databases.

VII. SUMMARY.

We have examined the concept of referential integrity in the context of relational schemas representing EER object structures. We have explored the effect of different relational representations of EER schemas on the structure of referential integrity constraints. Our analysis shows that the referential integrity concept can be simplified, and that the controversial referential integrity structures can be avoided without affecting the capability of relational schemas to represent EER object structures.

We have discussed the referential integrity capabilities of three relational database management systems (RDBMS), DB2, SYBASE, and INGRES. We have shown that some restrictions imposed by DB2 on the structure of referential integrity constraints limit the capability of defining in DB2 relational schemas representing EER object structures. We have compared the mechanisms provided by SYBASE and INGRES for the procedural specification of referential integrity constraints. We have shown that although conceptually similar, these mechanisms have technical differences, with the INGRES rule mechanism being more flexible and less restrictive than the SYBASE trigger mechanism.

Finally, we have pointed out the difficulty of using SYBASE triggers and INGRES rules. Using these mechanisms is labor-intensive and error-prone, therefore users should be insulated from them by a high-level interface that would allow non-procedural specifications of referential integrity constraints, and that would detect erroneous referential integrity structures. We have implemented such an interface as part of a *Schema Design* and *Translation* (*SDT*) tool [17]. ***SDT*** generates (i) abstract relational schemas from EER schemas, and (ii) RDBMS schema definitions from abstract relational schemas. Currently, ***SDT*** targets DB2, SYBASE, and INGRES. We have employed ***SDT*** for the generation of SYBASE and INGRES schema definitions from EER schemas consisting of approximately twenty object-sets. The difficulty of specifying SYBASE triggers and INGRES rules is illustrated by the amount of code (over one thousand lines) generated by ***SDT*** for the trigger and rule procedures involved in these definitions.

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APPENDIX

In this appendix we present two procedures, for mapping EER schemas into relational schemas, and for merging relation-schemes in relational schemas representing EER schemas.

A.1 Mapping EER Schemas.

The procedure for mapping EER schemas into relational schemas is based on procedures developed in [15]. We refer below to *generalization−sources*, which denote entity-sets that are not specializations of other entity-sets, and *independent* entity-sets, which denote generalization-sources that are not weak entity-sets. We assume that EER schemas satisfy certain *well−definedness* properties, such as the acyclicity of EER diagrams and the uniqueness of generalization-sources for specializations; these properties are discussed in [15] (see also [9] for a related discussion).

Definition – *Rmap*.

Input: a well-defined EER schema;

Output: a relational schema of the form $(R, F \cup I)$.

1. ***Value–Sets***. Every value-set is mapped into a relational domain.

2. ***Independent Entity–Sets***. An independent entity-set E_i is mapped into a relation-scheme, $R_i(X_i)$, such that: attribute set X_i is in a one-to-one correspondence with the EER attributes of E_i; every attribute A of X_i is assigned the domain corresponding to the value-set of the EER attribute of E_i corresponding to A. The subset Z_i of X_i that is in a one-to-one correspondence with the identifier of E_i, is specified as the primary-key of R_i, and key dependency R_i: $Z_i \rightarrow X_i$ is added to F.

3. ***Aggregation Object–Sets***. Let object-set O_i be the aggregation of (not necessarily distinct) object-sets O_{i_j}, $1 \leq j \leq m$, and let object-sets O_{i_j} correspond to relation-scheme $R_{i_j}(Y_{i_j})$, $1 \leq j \leq m$, respectively. Object-set O_i is mapped into relation-scheme $R_i(X_i)$, and inclusion dependencies $R_i[FK_{i_j}] \subseteq R_{i_j}[K_{i_j}]$, $1 \leq j \leq m$, are added to I. Attribute set X_i is the union of two disjoint sets of attributes, X'_i and X''_i, such that: (a) X'_i is in a one-to-one correspondence with the EER attributes of O_i, where the correspondence is specified as in (2) above; (b) $X''_i = \bigcup_{j=1}^{m} FK_{i_j}$, is a set of foreign-key attributes, where every foreign-key FK_{i_j} is in a one-to-one correspondence with primary-key K_{i_j}, $1 \leq j \leq m$, such that every attribute A of FK_{i_j} is assigned the domain associated with the attribute of K_{i_j} corresponding to A.

   If O_i is a ***weak entity-set*** and Z_i is the subset of X_i that is in a one-to-one correspondence with the identifier of E_i, then $Z_i X''_i$ is specified as the primary-key of R_i, and key dependency R_i: $Z_i X''_i \rightarrow X_i$ is added to F. If O_i is a ***relationship–set*** then if all the cardinalities of the object-sets involved in O_i are ***many***, then X''_i is specified as the primary-key of R_i, and key dependency $R_i : X''_i \rightarrow X_i$ is added to F; else $(X''_i - FK_{i_k})$ is specified as the primary-key of R_i, where FK_{i_k} is the ***foreign–key*** referencing the relation-scheme corresponding to an object-set that has cardinality ***one*** in O_i, and for every object-set O_{i_j} which has cardinality ***one*** in O_i, key dependency R_i: $(X''_i - FK_{i_j}) \rightarrow FK_{i_j}$ is added to F.

4. ***Specialization Entity–Sets***. Let entity-set E_i be the specialization of entity-sets E_{i_j}, $1 \leq j \leq m$, and E_k be the (unique) generalization-source of E_i. Let E_k correspond to relation-scheme $R_k(Y_k)$ and entity-sets E_{i_j} correspond to relation-schemes $R_{i_j}(Y_{i_j})$, $1 \leq j \leq m$, respectively. Entity-set E_i is mapped into relation-scheme $R_i(X_i)$, and inclusion dependencies $R_i[FK_{i_j}] \subseteq R_{i_j}[K_{i_j}]$, $1 \leq j \leq m$, are added to I. Attribute set X_i is the union of two disjoint sets of attributes, X'_i and X''_i, such that: (a) X'_i is in a one-to-one correspondence with the EER attributes of E_i, where the correspondence is specified as in (2) above; (b) X''_i is in a one-to-one correspondence with the primary-key of R_k, where the correspondence is specified as in (3.b) above; and (c) every foreign-key FK_{i_j}, $1 \leq j \leq m$, is equal to X''_i. X''_i is specified as the primary-key of R_i, and key dependency R_i: $X''_i \rightarrow X_i$ is added to F. ■

A.2 Merging Relations.

The merging procedure below is a restricted version of the procedure developed in [12]. In the definition of this procedure we use the relational algebra operations of ***total projection***, ***renaming***, and ***outer equi-join*** [10].

Let $R_i(X_i)$ be a relation-scheme associated with relation r_i, and W be a subset of X_i. The ***total projection*** of r_i on W is denoted $\pi\!\downarrow_W(r_i)$, and is equal to $\{t[W] \mid t \in r_i$ and t consists of non-null values$\}$.

Let $R_i(X_i)$, r_i, and W be defined as above, and let Y be an attribute set compatible with W. ***Renaming*** W to Y in r_i is denoted $rename(r_i; W \leftarrow Y)$, and is equal to $\{t' \mid t \in r_i,\ t'[X_i - W] = t[X_i - W],\ t'[Y] = t[W]\}$.

Let $R_i(X_i)$ and $R_j(X_j)$ be two relation-schemes associated with relations r_i and r_j, respectively; let Y and Z be two compatible and disjoint subsets of X_i and X_j, respectively; let k_i and k_j denote the number of attributes in X_i and X_j, respectively. We denote by ω a null value, and by ω^k a tuple consisting of k null values. The ***outer-equi-join*** of r_i and r_j on $(Y{=}Z)$ is denoted $r_i \overset{o}{\underset{Y=Z}{\bowtie}} r_j$, and is equal to the union of three relations: $r_1 = \{t \mid t[X_i] \in r_i,\ t[X_j] \in r_j,\ t[Y] = t[Z]\}$; $r_2 = \{t \mid t[X_i] = \omega^{k_i},\ t[X_j] \in r_j,\ \nexists\, t' \in r_i$ s.t. $t'[Y] = t[Z]\}$; and $r_3 = \{t \mid t[X_i] \in r_i,\ t[X_j] = \omega^{k_j},\ \nexists\, t'' \in r_j$ s.t. $t[Y] = t''[Z]\}$.

Definition – ***Rmerge***.

Input : a relational schema $RS = (R, F \cup I)$, and a subset of R, $\bar{R}$, such that

(a) the primary-keys of the relation-schemes of $\bar{R}$ are pairwise compatible;

(b) there exists a relation-scheme $R_p(X_p)$ in $\bar{R}$ such that for every R_i of $\bar{R}$, $i \neq p$, $R_i[K_i] \subseteq R_p[K_p] \in I$;

(c) every relation-scheme $R_i(X_i)$ of $\bar{R}$, $i \neq p$: (i) has exactly one non primary-key attribute; (ii) cannot be involved in the right-hand side of any inclusion dependency of I; and (iii) can be involved in the left-hand side of at most two inclusion dependencies, one involving R_p (see (b) above), and one of the form $R_i[X_i - K_i] \subseteq R_j[K_j]$.

Output: a relational schema $RS' = (R', F' \cup I')$.

Rmerge $(\bar{R})$ applied on RS generates RS' as follows:

1. R' results by replacing the relation-schemes of $\bar{R}$ in R with relation-scheme $R_m(X_m)$, such that $K_m := K_p$, $X_m := K_m \bigcup_{R_i(X_i) \in \bar{R}} (X_i - K_i)$, where the attributes of $(X_m - X_p)$ are allowed to have null values;
2. F' results by replacing in F all the key dependencies involving primary-keys associated with the relation-schemes of $\bar{R}$, with key dependency $R_m : K_m \rightarrow X_m$;
3. I' results by replacing R_i with R_m and K_i with K_m, in every inclusion dependency of I that involves a relation-scheme R_i of $\bar{R}$.

Rmerge $(\bar{R})$ is associated with two ***state mappings***, η and η', where η maps a state r of RS into a state r' of RS', and η' maps a state r' of RS' into a state $\tilde{r}$ of RS, as follows:

η is the ***identity*** for the relations of r that are associated with relation-schemes of $(R - \bar{R})$; and maps the set of relations $\{r_i \mid r_i \in r, r_i$ is associated with $R_i \in \bar{R}\}$ into r_m as follows:

(i) $r_m := r_p$; and

(ii) <u>for</u> each R_i of $(\bar{R} - \{R_p\})$

$$\underline{\text{do}} \quad r_m := \pi_{(X_m - K_i)}(r_m \mathop{\overset{\circ}{\bowtie}}_{K_m = K_i} r_i) \quad \underline{\text{enddo}}.$$

η' is the ***identity*** for every relation of $(r' - \{r'_m\})$; and maps relation r'_m of r' into relations $\tilde{r}_i$ as follows:

$$\tilde{r}_i := \textit{rename}(\, \pi\downarrow_{K_m(X_i - K_i)} (r'_m),\ K_m \leftarrow K_i),$$

where $R_i(X_i)$ is a relation-scheme of $\bar{R}$. ■