

# Security in Outsourcing of Association Rule Mining

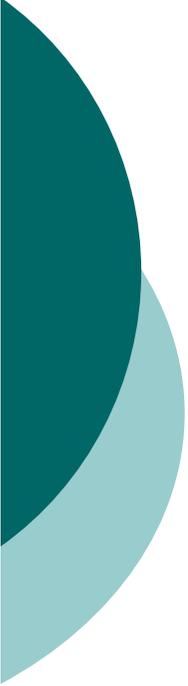
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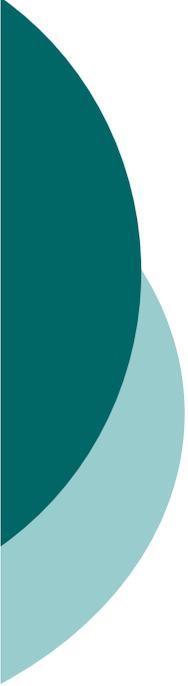
VLDB 2007, Vienna, Austria



# Agenda

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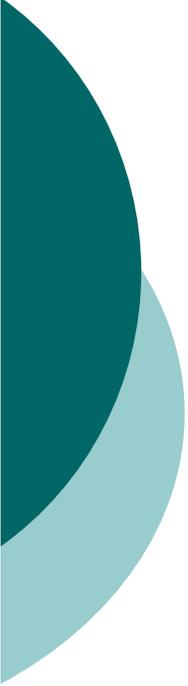
- Introduction and motivation
- Item mapping and encryption
- The algorithm for valid and complete transaction transformation
- Experiments
- Summary



# Introduction and motivation

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- Association rule mining
  - complexity of exponential order
- Motivation on outsourcing of mining task
  - lower cost
  - avoid hiring in-house specialists
  - consolidate data from different sources

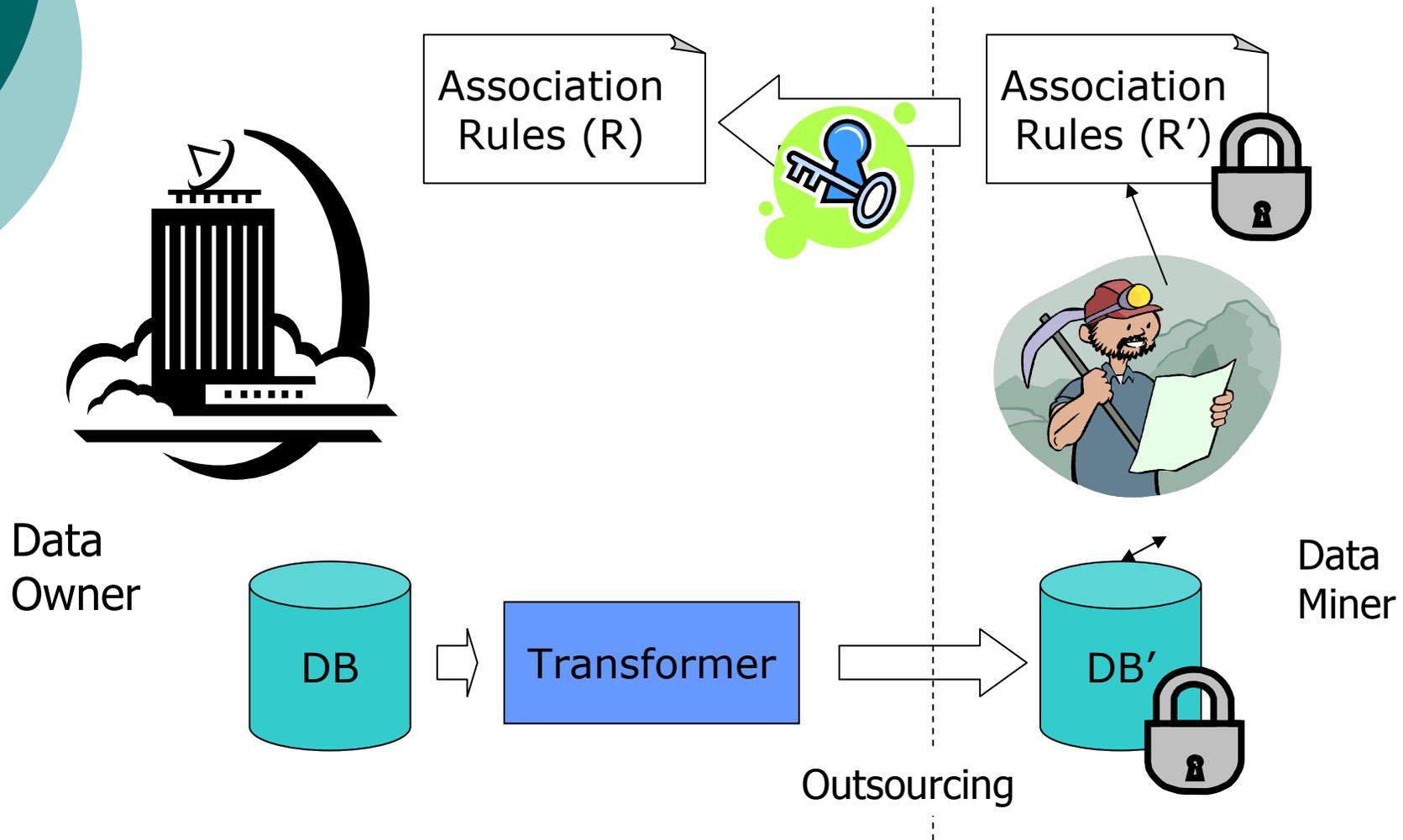


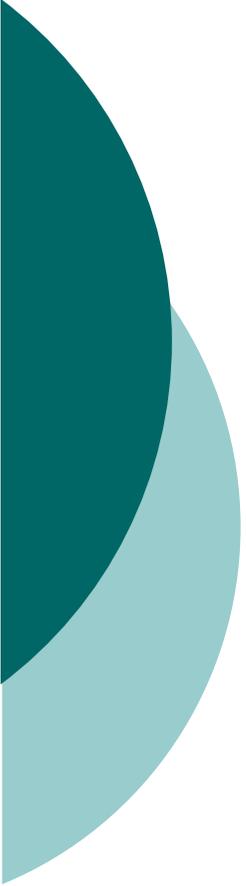
# Security concerns in outsourcing

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- The third party cannot be trusted
- Need to protect
  - Protect the input – prevent the miner (third party) to access the original transaction records
  - Protect the output – prevent the miner to see the “true” association rules

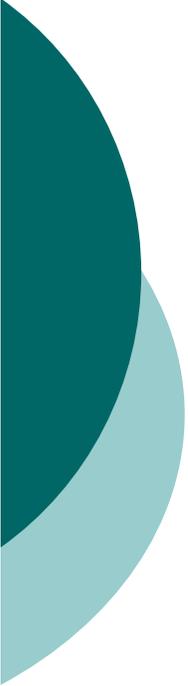
# Outsource model





# Item mapping - encryption

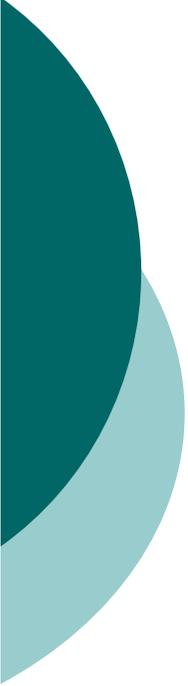
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## Example item mapping (one-to-one)

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- bread -> 54
- chocolate -> 165
  - <bread, chocolate> -> <165, 54>
- <54, 165> is large to the miner
  - <cheese, book> or <bread, chocolate>?
- **Similar to substitution cipher used in encryption of text**
- **Anything more secure ????**



# One-to-n item mapping

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- A one-to-n item mapping
  - B: a set of items
  - $m: I \rightarrow 2^B$
- Example,  $I = \{a, b, c\}$ ,  
 $B = \{1, 2, 3, 4, 5\}$ 
  - $m(a) = \{1, 4, 5\}$
  - $m(b) = \{2\}$
  - $m(c) = \{3, 5\}$
- Is one-to-n more secure ?



# Itemset mapping using one-to-n item mapping

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- $m: I \rightarrow 2^B$  : one-to-n item mapping
- $M: 2^I \rightarrow 2^B$  : itemset mapping

- $M(X) = \bigcup_{x \in X} m(x) = Y$

- $M^{-1}(Y) = X, \text{ if } M(X) = Y$

- Example:

- $M(\langle a, c \rangle) = \langle 1, 3, 4, 5 \rangle$

- $M(\langle b, c \rangle) = \langle 2, 3, 5 \rangle$

- $M^{-1}(\langle 1, 3, 4, 5 \rangle) = \langle a, c \rangle$

- $M^{-1}(\langle 1, 2, 3, 4, 5 \rangle) = \langle a, b, c \rangle$

$m:$
$a \rightarrow \{1, 4, 5\}$
$b \rightarrow \{2\}$
$c \rightarrow \{3, 5\}$

- Note:  $m$  is an item mapping,  $M$  is the itemset mapping

# Correctness – restrictions on one-to-n mapping

m:

a -> {1, 2}

b -> {2, 3}

c -> {1, 3}

- $\langle a, b \rangle \Rightarrow \langle 1, 2, 3 \rangle$
- $\langle a, b, c \rangle \Rightarrow \langle 1, 2, 3 \rangle$

**Collisions!**

**Decryption failure!**

m':

a -> {1, 2}

b -> {2, 3}

c -> {2, 4}

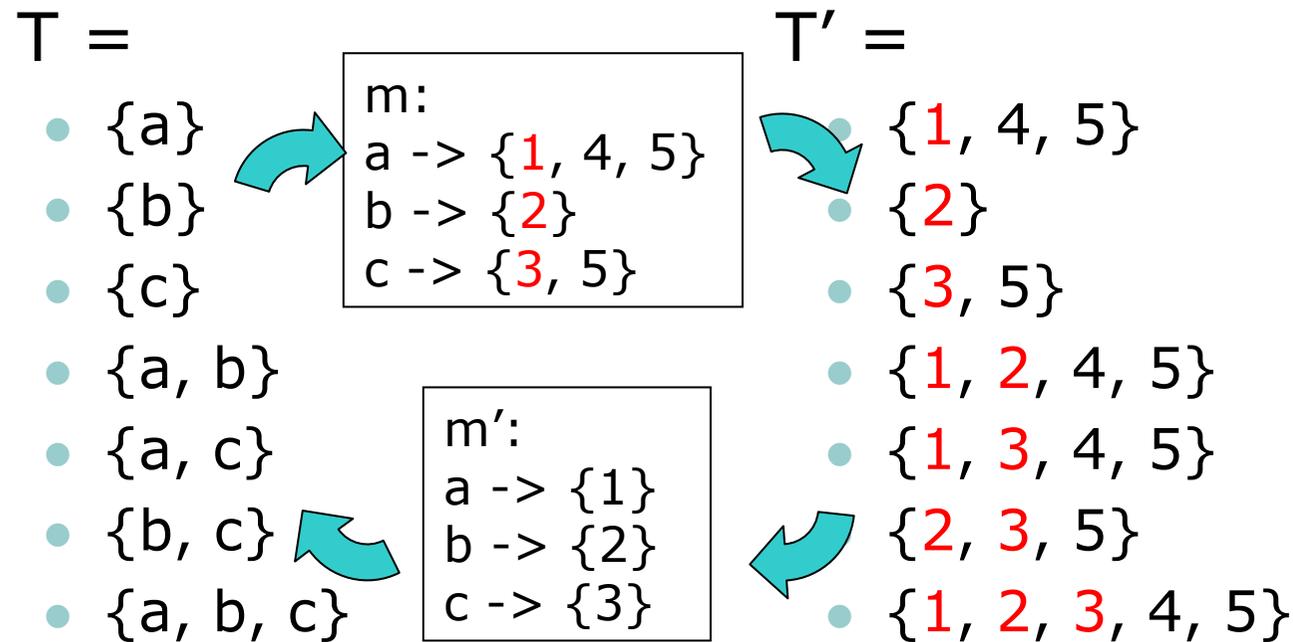
- $\langle a \rangle \Rightarrow \langle 1, 2 \rangle$
- ...
- $\langle a, b \rangle \Rightarrow \langle 1, 2, 3 \rangle$
- ...
- $\langle a, b, c \rangle \Rightarrow \langle 1, 2, 3, 4 \rangle$

Admissible Mapping : mapping of each item contains a unique item

**Result :  $M^{-1}(M(X)) = X$  (correct decryption) iff m is admissible**

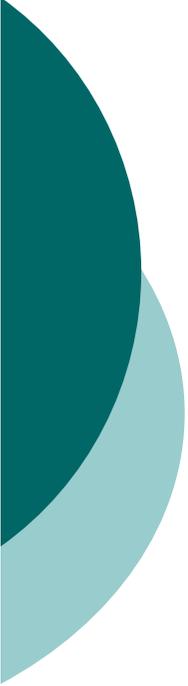
## Is one-to-n mapping more secure?

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To decrypt transactions encrypted by **m**, we can use **m'**!

(m is not more secure than m') !!!!



# Function coverage

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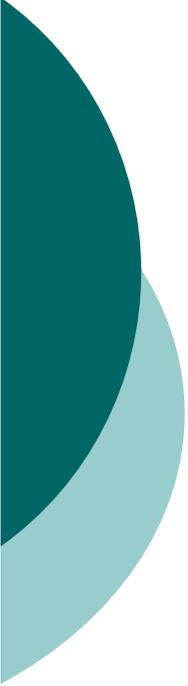
- $M_1: 2^I \rightarrow 2^{D1}$
- $M_2: 2^I \rightarrow 2^{D2}$
- $M_1$  covers  $M_2$  iff
  - for all  $X \in I$ , let  $Y = M_2(X)$ 
    - $M_2^{-1}(Y) = M_1^{-1}(Y \cap D1)$
- $M_1$  covers  $M_2$ 
  - If any transaction encrypted by  $M_2$  can be decrypted by using the inverse of  $M_1$



## One-to-n is not more secure than one-to-one mapping

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- Our results (proved)
  - Any admissible one-to-n itemset mapping **is covered by (can be decrypted by)** some one-to-one itemset mapping
- Bad news !!!
  - One-to-n item mapping is **NOT** more secure than a one-to-one item mapping



# One-to-n vs one-to-one

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- one-to-n vs one-to-one?
  - Intuitively, one-to-n should be more secure

Unfortunate Scenario:

- one-to-n + item mapping  
= one-to-one + item mapping

Our solution :

- Add a random component to transaction transformation
- It will make one-to-n always better (more secure) than one-to-one

# One-to-n Transformation

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- one-to-one mapping

- $a \rightarrow \{ 1 \}, b \rightarrow \{ 2 \}, \dots$
- $t = \{ a, b \} \rightarrow t' = \{ 1, 2 \}$

- one-to-n mapping

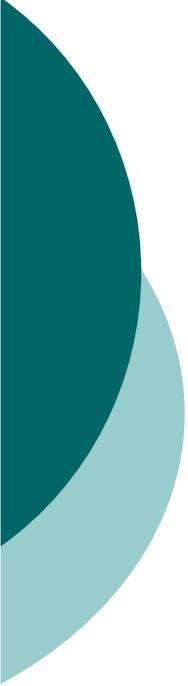
- $a \rightarrow \{ 1, 3 \}, b \rightarrow \{ 2, 3 \}, \dots$
- $t = \{ a, b \} \rightarrow t' = \{ 1, 2, 3 \}$

Randomly  
generated

- one-to-n transformation

- $a \rightarrow \{ 1, 3 \}, b \rightarrow \{ 2, 3 \}, \dots$
- $t = \{ a, b \} \rightarrow t' = \{ 1, 2, 3, \underline{4, 6} \}$



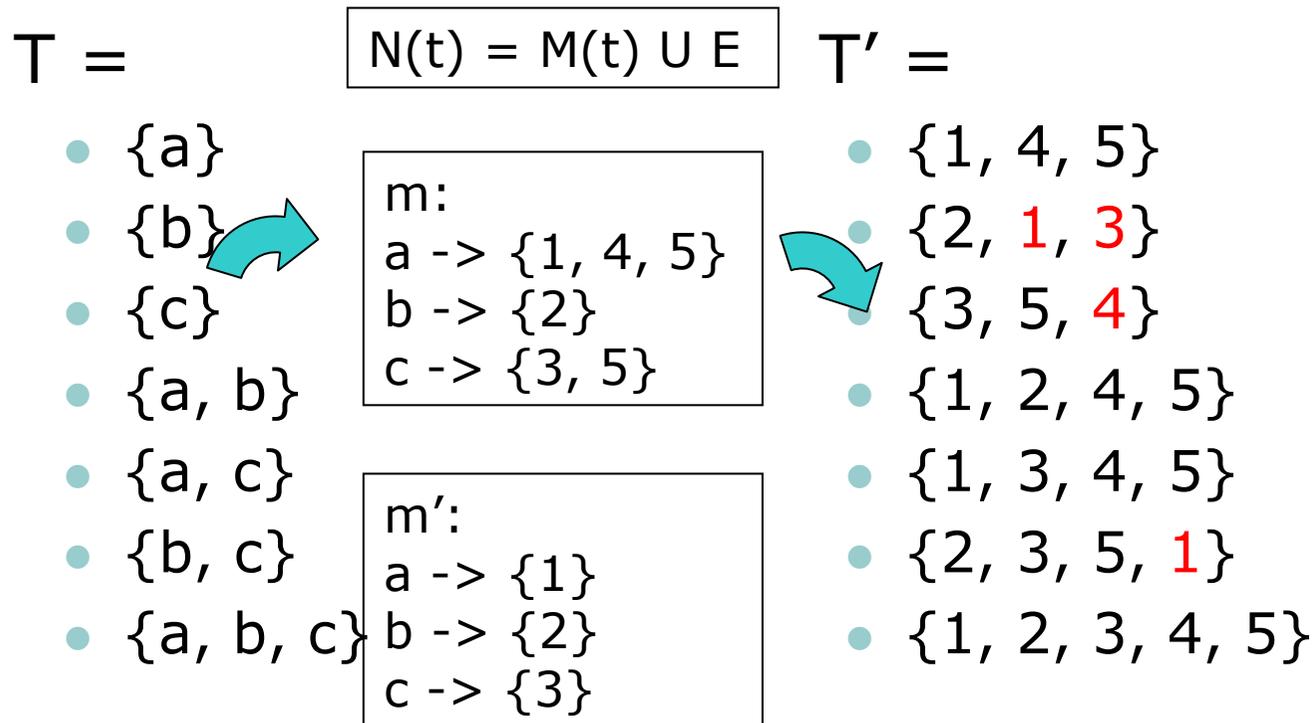


# Transaction transformation

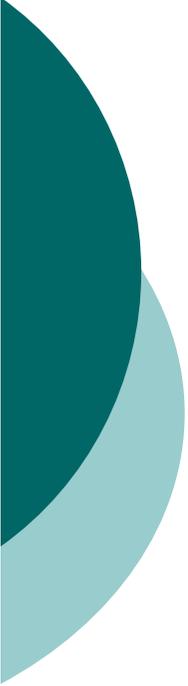
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- $M: 2^I \rightarrow 2^B$ , based on a one-to-n itemset mapping  $m$
- $N$ : transaction transformation
  - Maps from  $2^I$  to  $2^{B \cup F}$
- $t' = N(t) = M(t) \cup E$ 
  - $E$  is a **random** subset of  $B \cup F$ ;  $F$  is a set of items not in  $B$
- $N^{-1}(t') = \{x \mid m(x) \text{ in } t'\}$

# Example transformation



- The randomly inserted values does not affect the correctness of the decryption
- $m'$  can no longer be used to decrypt  $m$  !!



# Necessary properties of transformation N

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- Valid
  - The decryption is correct
  - $N^{-1}(N(t)) = t$
- Complete (based on valid)
  - For every transaction  $t$ ,  $N(t)$  generates every possible  $t'$  ( $= M(t) \cup E$ ) such that  $N^{-1}(t') = t$
- **Positive result : No one-to-one itemset mapping can cover a valid and complete transaction transformation from a one-to-n itemset mapping**

# Generating E for valid and complete transformation N

E = ?

$$N(\langle c \rangle) = \langle 3, 5 \rangle \cup E$$

- For m: a  $\rightarrow$  {1, 4, 5}

$$E = \{1\} \text{ or } \{4\}, \text{ but not } \{1, 4\}$$

- For m: b  $\rightarrow$  {2}

$$E = \Phi$$

- The transformation N is valid if E is either {1} or {4} or  $\Phi$  ;

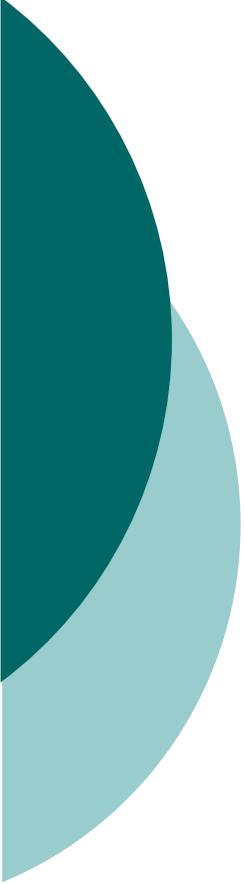
- N is complete if it is possible to generate all of the three cases, i.e.,  $E = \{1\}$  or  $\{4\}$  or  $\Phi$ .

m:

a  $\rightarrow$  {1, 4, 5}

b  $\rightarrow$  {2}

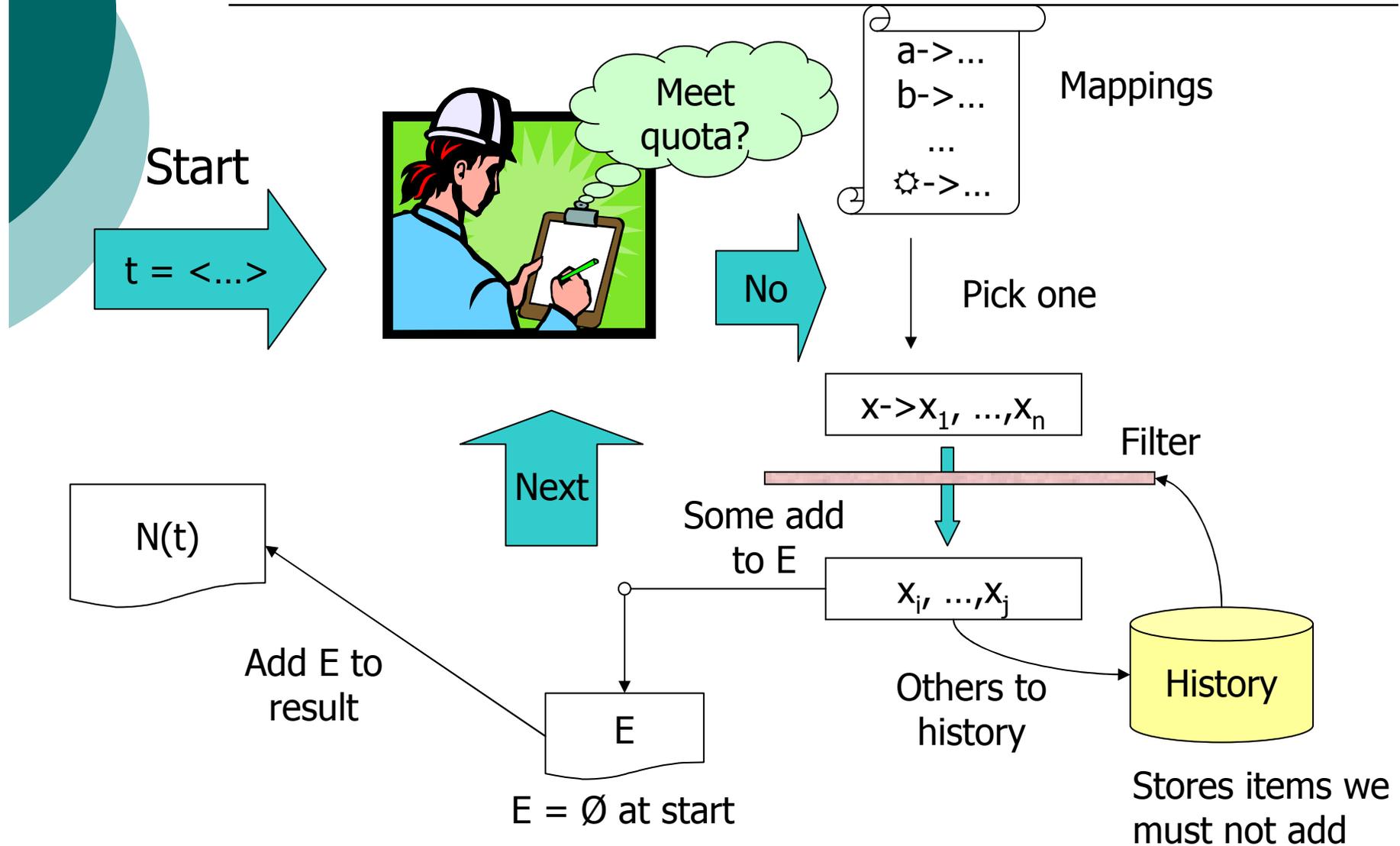
c  $\rightarrow$  {3, 5}

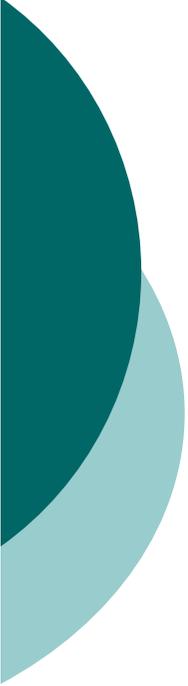


# Algorithm – valid and complete transaction transformation

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# Algorithm to perform valid and complete transformation

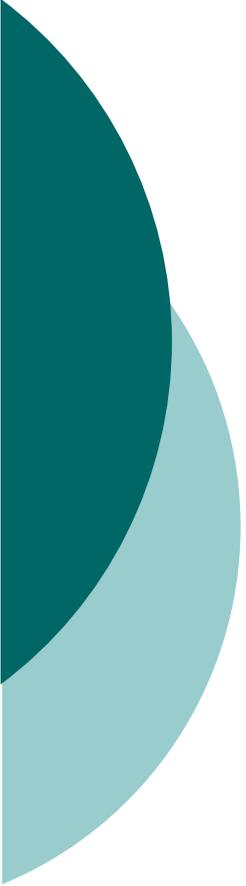




## Important Property

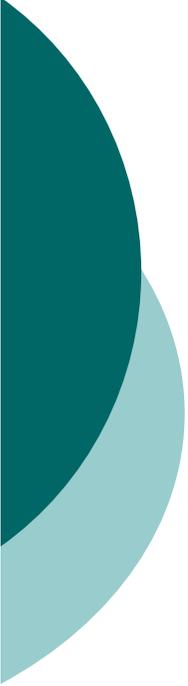
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- The transaction transformation produced by the Algorithm is valid and complete.



# Experiments

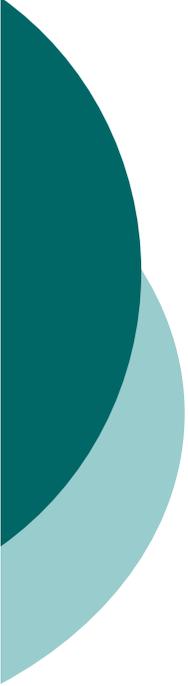
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# Design

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- Purpose
  - Study security and efficiency of the model
- Security
  - Assume the attacker gets the relative frequencies
  - Implemented genetic algorithm for frequency analysis
- Efficiency
  - Transformation time vs mining time
  - Overhead at the miner side



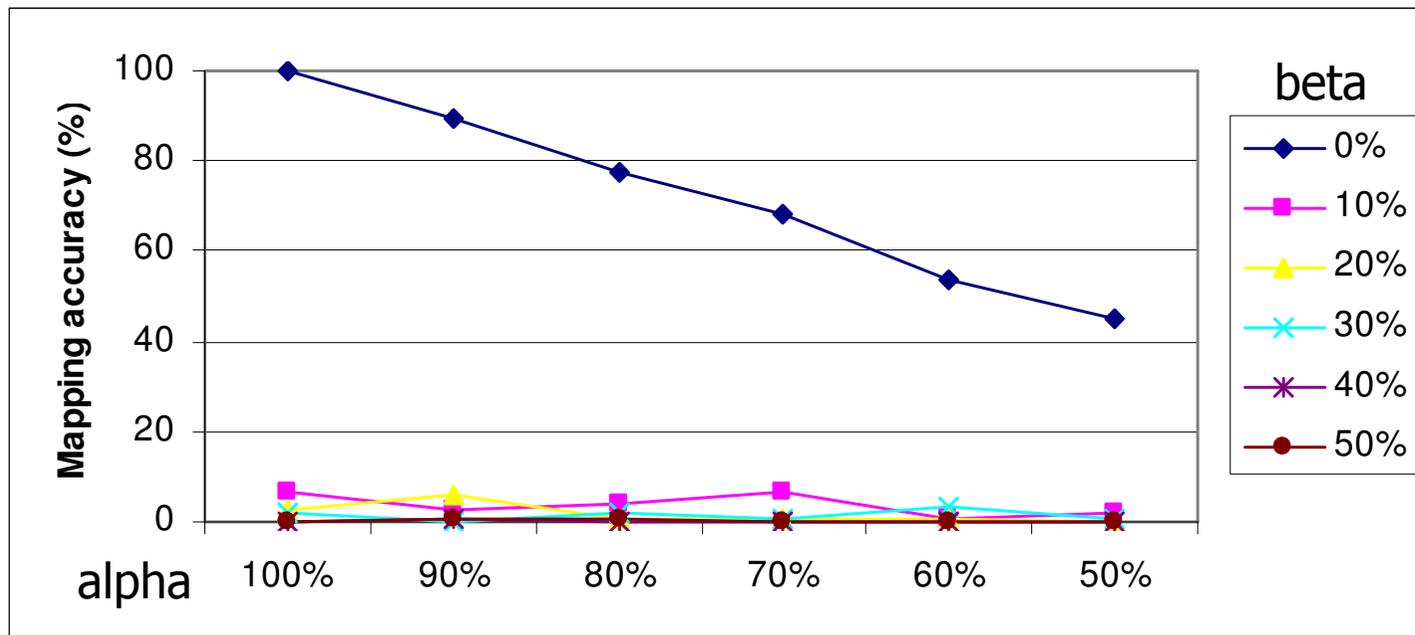
# Background knowledge

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- Purpose: simulate a real attacker in practice
- Where does the attacker get knowledge?  
(Assumption)
  - In many cases, the statistics of the global industry is public (**background knowledge**)
- Background Knowledge (with two parameters)
  - alpha: knows alpha% of large itemsets in original database
  - beta: the support in the knowledge is in the range
    - real support \*  $(1 \pm \text{beta})$

# Mapping accuracy

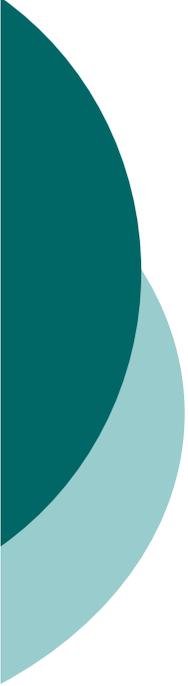
- Measure how many mapping is correct
  - Only measure those in background knowledge since there is no info for other mappings



# Efficiency

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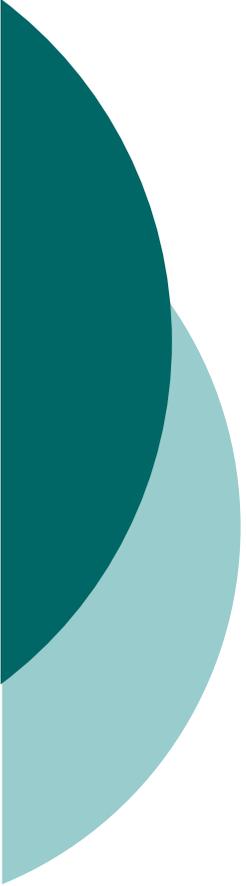
	100k	200k	300k	400k	500k
Cost at owner side (transformation and recovery)	2.8s	5.5s	9.5s	11.2s	12.5s
Cost at miner side	195s	488s	738s	945s	1122s
Original mining cost	80s	204s	293s	383s	465s



## Summary

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- The idea of substitution cipher is used in the problem of encryption of transaction database
- One-to-n item mapping cannot be directly applied since it is effectively a one-to-one item mapping
- Transaction transformation is proposed and shown to be valid and complete
- Experiments show that it is suitable for outsourcing



End

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