Analysis of Two Existing and One New Dynamic Programming Algorithm for the Generation of Optimal Bushy Join Trees without Cross Products

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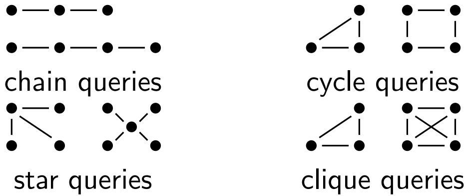
Overview

1. Motivation
2. Existing Algorithms: DPsize, DPsub
3. Idea
4. Our Algorithm: DPccp
5. Evaluation
6. Conclusion

Motivation

Problem: Generate the best bushy join tree not containing a cross product.



- structure of query graph greatly affects complexity
- e.g. cliques are NP hard in general, chains are in $O(n^3)$
- algorithm should adapt to the graph structure

Motivation - Dynamic Programming Strategies

Advantages:

- general purpose, many cost functions
- find the optimal solution

Basic scheme:

- solve problems only once
- build solutions from smaller solutions
- here: join pairs of optimal join trees
- main difference between strategies: enumeration order

query graph structure should affect enumeration order

Existing Algorithms - DPsize

- organize DP by the size of the join tree
- enumerate ordered by the number of joined relations
- first all with 2 relations, with 3 relations, etc.
- for a given size n consider all L, R such that $n = |L| + |R|$
- prune pairs afterwards (connectedness, disjointness, costs)
- problem: only few DP slots, many pairs considered

good algorithm for chains, very bad for cliques:

	chains	cycles	stars	cliques
pairs	$O(n^4)$	$O(n^4)$	$O(4^n)$	$O(4^n)$

absolute complexity also interesting, see the paper

Existing Algorithms - DPsub

- organize DP by the set of relations involved
- enumerate subsets before supersets
- first $\{R_1\}$, then $\{R_2\}$, then $\{R_1, R_2\}$ etc.
- for a given problem P consider all L,R such that $P = L \cup R, L \cap R = \emptyset$
- prune pairs afterwards (connectedness, costs)
- problem: always 2^n DP slots, fixed enumeration

good algorithm for cliques, but adapts badly:

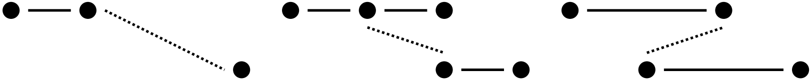
	chains	cycles	stars	cliques
pairs	$O(2^n)$	$O(n2^n)$	$O(3^n)$	$O(3^n)$

faster than DPsize for stars and cliques, slower for chains and cycles.

Idea - Observation

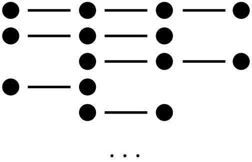
DPsize and DPsub generate many pairs that are pruned anyway (connectedness, overlap).

Typical pruned pairs (chain with 4 relations):



not connected — not disjoint — invalid subproblems

last example ⇒ every join partner must be a connected subgraph:



Idea - New Approach

- reformulation as graph theoretic problem:
- enumerate all connected subgraphs of the query graph
- for each subgraph enumerate all other connected subgraphs that are disjoint but connected to it
- each connected subgraph - complement pair (ccp) can be joined
- enumerate them suitable for DP $\Rightarrow$ DP algorithm

algorithm adapts naturally to the graph structure:

	chains	cycles	stars	cliques
pairs	$O(n^3)$	$O(n^3)$	$O(n2^n)$	$O(3^n)$

Lohman et al: #ccp is a lower bound for all DP enumeration algorithms

Idea - Effect on Search Space

Absolute number of generated pairs

	Chain			Star		
n	#ccp	DPsub	DPsize	#ccp	DPsub	DPsize
2	1	2	1	1	2	1
5	20	84	73	32	130	110
10	165	3,962	1,135	2,304	38,342	57,888
15	560	130,798	5,628	114,688	9,533,170	57,305,929
20	1,330	4,193,840	17,545	4,980,736	2,323,474,358	59,892,991,338
	Cycle			Clique		
n	#ccp	DPsub	DPsize	#ccp	DPsub	DPsize
2	1	2	1	1	2	1
5	40	140	120	90	180	280
10	405	11,062	2,225	28,501	57,002	306,991
15	1,470	523,836	11,760	7,141,686	14,283,372	307,173,877
20	3,610	22,019,294	37,900	1,742,343,625	3,484,687,250	309,338,182,241

New Algorithm

- two steps: enumerate all connected subgraphs, enumerate disjoint but connected subgraphs for a given one $\Rightarrow$ pairs
- enumerate all pairs, enumerate no duplicates, enumerate for DP
- if (a, b) is enumerated, do not enumerate (b, a)
- requires total ordering of connected subgraphs
- preparation: label nodes breadth-first from 0 to $n-1$

Preliminaries, given query graph $G = (V, E)$:

$$\begin{aligned} V &= \{v_0, \ldots, v_{n-1}\} \\ \mathcal{N}(V') &= \{v' | v \in V' \wedge (v, v') \in E\} \\ \mathcal{B}_i &= \{v_j | i \leq i\} \end{aligned}$$

New Algorithm - Connected Subgraphs

EnumerateCsg(G)
for all $i \in [n-1, \dots, 0]$ **descending** {
    emit $\{v_i\}$;
    EnumerateCsgRec(G, $\{v_i\}$, $\mathcal{B}_i$);
}

EnumerateCsgRec(G, S, X)
$N = \mathcal{N}(S) \setminus X$;
for all $S' \subseteq N$, $S' \neq \emptyset$, enumerate subsets first {
    emit $(S \cup S')$;
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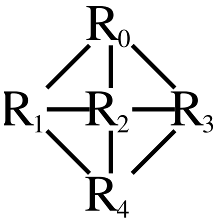

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Choose all nodes as enumeration start node once

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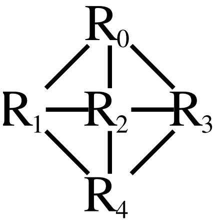

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}

First emit only the node itself as subgraph

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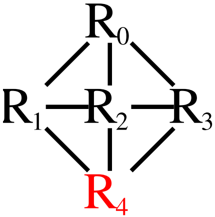

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Then enlarge the subgraph recursively

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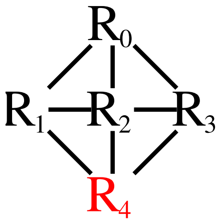

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}

Prohibit nodes with smaller labels. Thus the set of valid nodes increases over time

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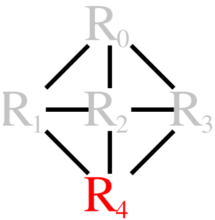

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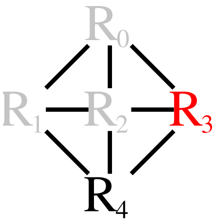

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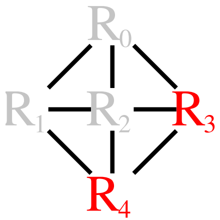

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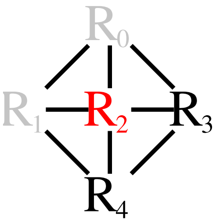

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In each recursion, find all neighboring nodes that are not prohibited

EnumerateCsgRec(G, S, X)
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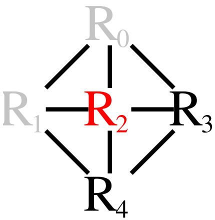

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Add all combinations to the subgraph and emit the new subgraph

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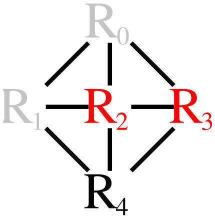

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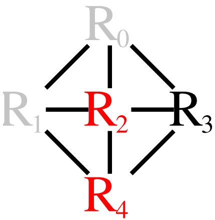

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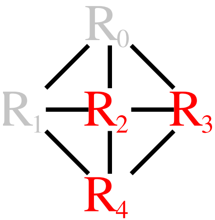

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for all $i \in [n-1, \ldots, 0]$ **descending** {
    emit $\{v_i\}$;
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}

Then, add all combinations to the subgraph and increase recursively

EnumerateCsgRec(G, S, X)
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for all $S' \subseteq N$, $S' \neq \emptyset$, enumerate subsets first {
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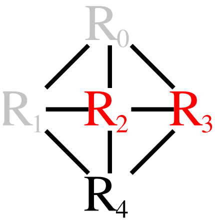

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The neighborhood is prohibited during recursion, preventing duplicates

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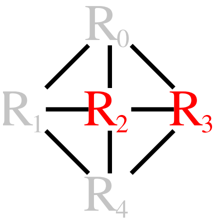

New Algorithm - Complementary Subgraphs

EnumerateCmp(G,S_1)
$X = \mathcal{B}_{\min(S_1)} \cup S_1$;
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for all ($v_i \in N$ by descending i) {
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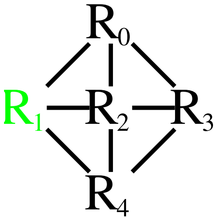

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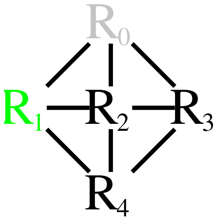
}

Prohibit all nodes that will be start nodes later on and the primary subgraph



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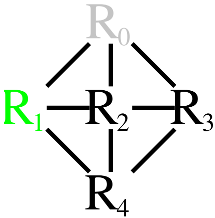
Find all neighboring nodes that are not prohibited



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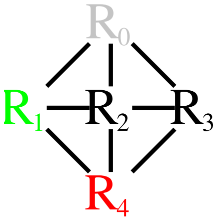
Consider each of the nodes



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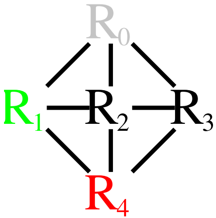
}

Choose the node as complementary subgraph and emit it



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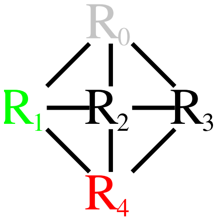
}

Recursively increase the subgraph re-using EnumerateCsgRec



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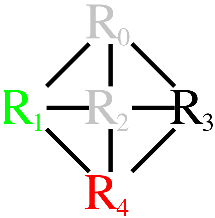
Again prohibit nodes with a smaller label to prevent duplicates



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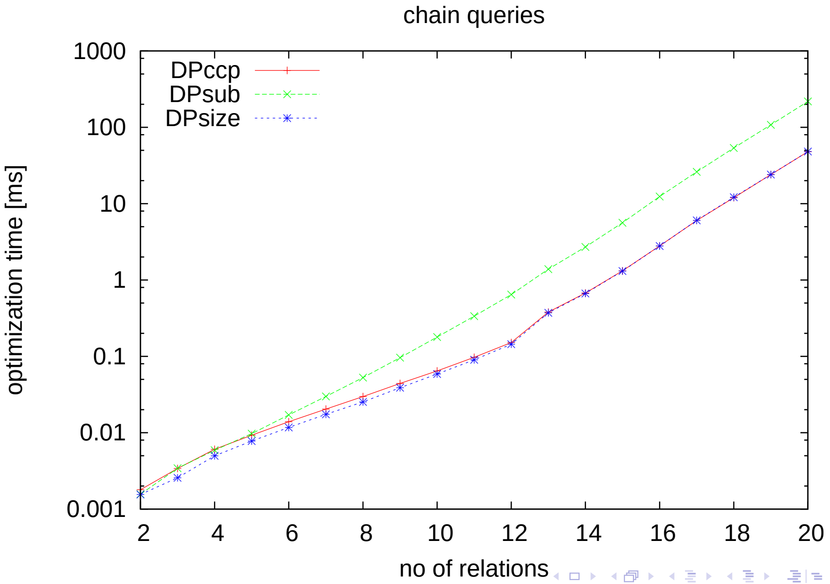
}

- EnumerateCsg+EnumerateCmp produce all ccp
- resulting algorithm DPccp considers exactly #ccp pairs
- which is the lower bound for all DP enumeration algorithms
- formal proof of correctness in the paper

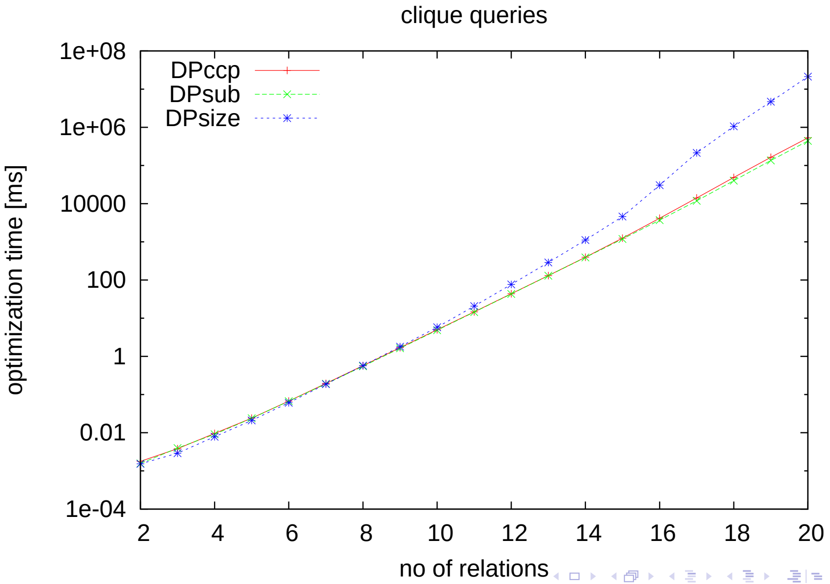
Evaluation

- asymptotically DPccg is clearly superior
- but implementation is more involved
- measure overhead by comparing runtime
- extremes: chain (favors DPsize) and clique (favors DPsub)
- in between: stars, show effect of search space reduction
- real queries will also be between chains and cliques

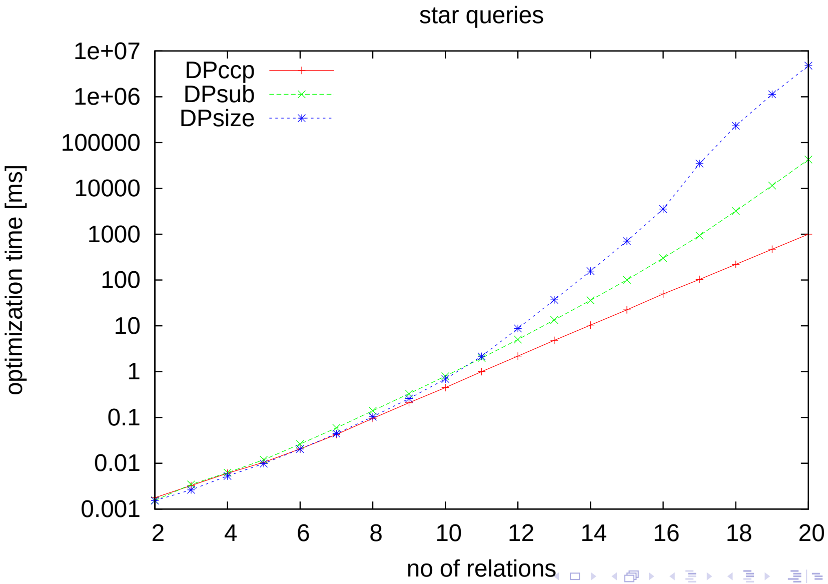
Evaluation - Chains



Evaluation - Cliques



Evaluation - Stars



Conclusion

- analytic and experimental evaluation of DPsize/DPsub
- DPsize is superior for chains/cycles
- DPsub is superior for stars/cliques
- new algorithm DPccg adopts to query graph structure
- minimal number of pairs
- low implementation overhead
- DPccp is the DP algorithm to choose

Number of Connected Subgraphs

	chains	cycles	stars	cliques
#csg	$O(n^2)$	$O(n^2)$	$O(2^n)$	$O(2^n)$

- determines the size of the DP table
- determines the number of cardinality estimations
- much less than #ccp