Blogel: A Block-Centric Framework for Distributed Computation on Real-World Graphs

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ABSTRACT

The rapid growth in the volume of many real-world graphs (e.g., social networks, web graphs, and spatial networks) has led to the development of various vertex-centric distributed graph computing systems in recent years. However, real-world graphs from different domains have very different characteristics, which often create bottlenecks in vertex-centric parallel graph computation. We identify three such important characteristics from a wide spectrum of real-world graphs, namely (1)skewed degree distribution, (2)large diameter, and (3)(relatively) high density. Among them, only (1) has been studied by existing systems, but many real-world power-law graphs also exhibit the characteristics of (2) and (3). In this paper, we propose a block-centric framework, called Blogel, which naturally handles all the three adverse graph characteristics. Blogel programmers may think like a block and develop efficient algorithms for various graph problems. We propose parallel algorithms to partition an arbitrary graph into blocks efficiently, and block-centric programs are then run over these blocks. Our experiments on large real-world graphs verified that Blogel is able to achieve orders of magnitude performance improvements over the state-of-the-art distributed graph computing systems.

1. INTRODUCTION

Due to the growing need to deal with massive graphs in various graph analytic and graph mining applications, many distributed graph computing systems have emerged in recent years, including Pregel [11], GraphLab [10], PowerGraph [4], Giraph [1], GPS [15], and Mizan [8]. Most of these systems adopt the *vertex-centric* model proposed in [11], which promotes the philosophy of "thinking like a vertex" that makes the design of distributed graph algorithms more natural and easier. However, the vertex-centric model has largely ignored the characteristics of real-world graphs in its design and can hence suffer from severe performance problems.

We investigated a broad spectrum of real-world graphs and identified three characteristics of large real-world graphs, namely (1)*skewed degree distribution* (common for power-law and scale free graphs such as social networks and web graphs), (2)*(relatively) high density* (common for social networks, mobile phone networks, SMS networks, some web graphs, and the cores of most large graphs), and (3)*large diameter* (common for road networks, terrain graphs, and some large web graphs). These three characteristics are particularly adverse to vertex-centric parallelization as they are often the major cause(s) to one or more of the following three performance bottlenecks: *skewed workload distribution*, *heavy message passing*, and *impractically many rounds of computation*.

Let us first examine the performance bottleneck created by skewed degree distribution. The vertex-centric model assigns each vertex together with its adjacency list to a machine, but neglects the difference in the number of neighbors among different vertices. As a result, for graphs with skewed degree distribution, it creates unbalanced workload distribution that leads to a long elapsed running time due to waiting for the last worker to complete its job. For example, the maximum degree of the BTC RDF graph used in our experiments is 1,637,619, and thus a machine holding such a high-degree vertex needs to process many incoming messages and send out many outgoing messages to its neighbors, causing imbalanced workload among different machines.

Some existing systems proposed techniques for better load balancing [4, 15], but they do not reduce the overall workload. However, for many real-world graphs including power-law graphs such as social networks and mobile phone networks, the average vertex degree is large. Also, most large real-world graphs have a high-density core (e.g., the k-core [16] and k-truss [19] of these graphs). Higher density implies heavier message passing for vertex-centric systems. We show that heavy communication workload due to high density can also be eliminated by our new computing model.

For processing graphs with a large diameter δ, the message (or neighbor) propagation paradigm of the vertex-centric model often leads to algorithms that require $O(\delta)$ rounds (also called supersteps) of computation. For example, a single-source shortest path algorithm in [11] takes 10,789 supersteps on a USA road network. Apart from spatial networks, some large web graphs also have large diameters (from a few hundred to thousands). For example, the vertex-centric system in [14] takes 2,450 rounds for computing strongly connected components on a web graph.

To address the performance bottlenecks created by real-world graphs in vertex-centric systems, we propose a *block-centric* graph-parallel abstraction, called **Blogel**. Blogel is conceptually as simple as Pregel but works in coarser-grained graph units called *blocks*. Here, a block refers to a connected subgraph of the graph, and message exchanges occur among blocks.

Blogel naturally addresses the problem of skewed degree distribution since most (or all) of the neighbors of a high-degree vertex v are inside v's block, and they are processed by sequential in-memory algorithms without any message passing. Blogel also solves the heavy communication problem caused by high density,

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since the neighbors of many vertices are now within the same block, and hence they do not need to send/receive messages to/from each other. Finally, Blogel effectively handles large-diameter graphs, since messages now propagate in the much larger unit of blocks instead of single vertices, and thus the number of rounds is significantly reduced. Also, since the number of blocks is usually orders of magnitude smaller than the number of vertices, the workload of a worker is significantly less than that of a vertex-centric algorithm.

A central issue to Blogel is whether we can partition an arbitrary input graph into blocks efficiently. We propose a *graph Voronoi diagram based partitioner* which is a fully distributed algorithm, while we also develop more effective partitioners for graphs with additional information available. Our experiments show that our partitioning algorithms are efficient and effective.

We present a user-friendly and flexible programming interface for Blogel, and illustrate that programming in Blogel is easy by designing algorithms for a number of classic graph problems in the Blogel framework. Our experiments on large real-world graphs with up to hundreds of millions of vertices and billions of edges, and with different characteristics, verify that our block-centric system is orders of magnitude faster than the state-of-the-art vertex-centric systems [1, 10, 4]. We also demonstrate that Blogel can effectively address the performance bottlenecks caused by the three adverse characteristics of real-world graphs.

The rest of the paper is organized as follows. Section 2 reviews related systems. Section 3 illustrates the merits of block-centric computing. Section 4 gives an overview of the Blogel framework, Section 5 presents Blogel's programming interface, and Section 6 discusses algorithm design in Blogel. We describe our partitioners in Section 7 and present performance results in Section 8. Finally, we conclude our paper in Section 9.

2. BACKGROUND AND RELATED WORK

We first define some basic graph notations and discuss the storage of a graph in distributed systems.

Notations. Given an undirect graph $G = (V, E)$, we denote the neighbors of a vertex $v \in V$ by $\Gamma(v)$; if G is directed, we denote the in-neighbors (out-neighbors) of v by $\Gamma_{in}(v)$ ($\Gamma_{out}(v)$). Each $v \in V$ has a unique integer ID, denoted by $id(v)$. The diameter of G is denoted by $\delta(G)$, or simply δ when G is clear from the context.

Graph storage. We consider a shared-nothing architecture where data is stored in a *distributed file system (DFS)*, such as *Hadoop's DFS (HDFS)*. We assume that a graph is stored as a distributed file in HDFS, where each line records a vertex and its adjacency list. A distributed graph computing system involves a cluster of k workers, where each worker w_i holds and processes a batch of vertices in its main memory. Here, "worker" is a general term for a computing unit, and a machine can have multiple workers in the form of threads/processes. A job is processed by a graph computing system in three phases: **(1)loading**: each worker loads a portion of vertices from HDFS into main-memory; the workers then exchange vertices through the network (by hashing over vertex ID) so that each worker w_i finally holds all and only those vertices assigned to w_i; **(2)iterative computing**: in each iteration, each worker processes its own portion of vertices sequentially, while different workers run in parallel and exchange messages. **(3)dumping**: each worker writes the output of all its processed vertices to HDFS. Most existing graph-parallel systems follow this procedure.

Pregel's computing model. Pregel [11] is designed based on the bulk synchronous parallel (BSP) model. It distributes vertices to workers, where each vertex is associated with its adjacency list. A program in Pregel implements the *compute*() function and proceeds in iterations (called *supersteps*). In each superstep, the program calls *compute*() for each active vertex. The *compute*() function performs the user-specified task for a vertex v, such as processing v's incoming messages (sent in the previous superstep), sending messages to other vertices (for the next superstep), and making v vote to halt. A halted vertex is reactivated if it receives a message in a subsequent superstep. The program terminates when all vertices vote to halt and there is no pending message for the next superstep.

Pregel also supports message combiner. For example, if there are k numerical messages in worker w_i to be sent to a vertex v in worker w_j and suppose that only the sum of the message values matters, then user can implement a *combine*() function to sum up the message values and deliver only the sum to v in w_j, thus reducing the number of messages to be buffered and transmitted. Pregel also supports aggregator, which is useful for global communication. Each vertex can provide a value to an aggregator in *compute*() in a superstep. The system aggregates those values and makes the aggregated result available to all vertices in the next superstep.

Since Google's Pregel is proprietary, many open-source systems have been developed based on Pregel's computing model, such as Giraph [1] and GPS [15]. In particular, GPS uses a technique called large adjacency list partitioning to handle high-degree vertices.

Vertex placement. Vertex placement rules that are more sophisticated than hashing were studied in [17], which aims at minimizing the number of cross-worker edges while ensuring that workers hold approximately the same number of vertices. However, their method requires expensive preprocessing but the performance gain is limited (e.g., only a speed up of 18%–39% for running PageRank).

Giraph++. A recent system, Giraph++ [18], proposed a graph-centric programming paradigm that opens up the block structure to users. However, Giraph++'s programming paradigm is still vertex-centric since it does not allow a block to have its own states like a vertex. Instead, each block is treated as two sets of vertices, internal ones and boundary ones. As such, Giraph++ does not support block-level communication, i.e., message passing is still from vertices to vertices, rather than from blocks to blocks which is more efficient for solving some graph problems. For example, an algorithm for computing connected components is used to demonstrate Giraph++ programming in [18], in which vertex IDs are passed among internal vertices and boundary vertices. We implement an algorithm for the same problem in our block-centric framework in Section 3, and show that it is much simpler and more efficient to simply exchange block IDs between blocks directly. Moreover, since Giraph++ is Java-based, the intra-block computation inevitably incurs (de)serialization cost for each vertex accessed; in contrast, the intra-block computation in Blogel is simply a main-memory algorithm without any additional cost. Finally, Giraph++ extends METIS [7] for graph partitioning, which is an expensive method. In Section 8, we will show that graph partitioning and computing is much more efficient in Blogel than in Giraph++.

GRACE. Another recent system, GRACE [20], which works in a single-machine environment, also applies graph partitioning to improve performance. GRACE enhances vertex-centric computing with a scheduler that controls the order of vertex computation in a block. This can be regarded as a special case of block-centric computing, where the intra-block computing logic is totally defined by the scheduler. Although this model relieves users of the burden of implementing the intra-block computing logic, it is not as expressive as our block-centric paradigm. For example, GRACE does not allow a block to have its own states and does not support block-level communication. We remark that Blogel and GRACE have different focuses: GRACE attempts to improve main memory bandwidth utilization in a single-machine environment, while Blo-

gel aims to reduce the computation and communication workload in a distributed environment.

GraphLab. Unlike Pregel's synchronous model and message passing paradigm, GraphLab [10] adopts a shared memory abstraction and supports asynchronous execution. It hides the communication details from programmers: when processing a vertex, users can directly access its own field as well as the fields of its adjacent edges and vertices. GraphLab maintains a global scheduler, and workers fetch vertices from the scheduler for processing, possibly adding the neighbors of these vertices into the scheduler. Asynchronous computing may decrease the workload for some algorithms but incurs extra cost due to data locking/unlocking. A recent version of GraphLab, called PowerGraph [4], partitions the graph by edges instead of by vertices, so that the edges of a high-degree vertex are handled by multiple workers. Accordingly, an edge-centric Gather-Apply-Scatter (GAS) computing model is used. However, GraphLab is less expressive than Pregel, since a vertex cannot communicate with a non-neighbor, and graph mutation is not supported.

3. MERITS OF BLOCK-CENTRIC COMPUTING: A FIRST EXAMPLE

We first use an example to illustrate the main differences in the programming and performance between the block-centric model and vertex-centric model, by considering the Hash-Min algorithm of [13] for finding connected components (CCs). We will show the merits of the block-centric model for processing large real-world graphs with the three characteristics discussed in Section 1.

Given a CC C, we denote the set of vertices of C by $V(C)$, and define the ID of a CC C to be $cc(v) = \min\{id(u) : u \in V(C)\}$. Hash-Min computes $cc(v)$ for each vertex $v \in V$. The idea is to broadcast the smallest vertex ID seen so far by each vertex v, denoted by $min(v)$.

For the vertex-centric model, in superstep 1, each vertex v sets $min(v)$ to be the smallest ID among $id(v)$ and $id(u)$ of all $u \in \Gamma(v)$, broadcasts $min(v)$ to all its neighbors, and votes to halt. In each later superstep, each vertex v receives messages from its neighbors; let min^* be the smallest ID received, if $min^* < min(v)$, v sets $min(v) = min^*$ and broadcasts min^* to its neighbors. All vertices vote to halt at the end of a superstep. When the process converges, $min(v) = cc(v)$ for all v.

Next we discuss the implementation of Hash-Min in the block-centric framework. Let us assume that vertices are already grouped into blocks by our partitioners (to be discussed in Section 7), which guarantee that vertices in a block are connected. Each vertex belongs to a unique block, and let $block(v)$ be the ID of the block that v belongs to. Let $\mathbb{B}$ be the set of blocks, and for each block $B \in \mathbb{B}$, let $V(B)$ be the set of vertices of B, and $id(B)$ be the integer ID of B. We define $\Gamma(B)=\bigcup_{v \in V(B)}\{block(u) : u \in \Gamma(v)\}$. Thus, we obtain a *block-level graph*, $\mathbb{G} = (\mathbb{B}, \mathbb{E})$, where $\mathbb{E} = \{(B_i, B_j) : B_i \in \mathbb{B}, B_j \in \Gamma(B_i)\}$. We then simply run Hash-Min on $\mathbb{G}$ where blocks in $\mathbb{B}$ broadcast the smallest block ID that they have seen. Similar to the vertex-centric algorithm, each block B maintains a field $min(B)$, and when the algorithm converges, all vertices v with the same $min(block(v))$ belong to one CC.

We now analyze why the block-centric model is more appealing than the vertex-centric model. First, we consider the processing of graphs with skewed degree distribution, where we compare G with $\mathbb{G}$. Most real-world graphs with skewed degree distribution consists of a giant CC and many small CCs. In this case, our partitioner computes a set of roughly even-sized blocks from the giant CC, while each small CC forms a block. Let b be the average number of vertices in a block and d_{max} be the maximum vertex degree in G. The vertex-centric model works on G and hence a high-degree vertex may send/receive $O(d_{max})$ messages each round, causing skewed workload among the workers. On the contrary, the block-centric model works on $\mathbb{G}$ and a high-degree vertex involves at most $O(n/b)$ messages each round, where n is the number of vertices in the giant CC. For power-law graphs, d_{max} can approach n and n/b can be a few orders of magnitude smaller than d_{max} for a reasonable setting of b (e.g., $b = 1000$).

		Computing Time	Total Msg #	Superstep #
BTC	**Vertex-Centric**	28.48 s	1,188,832,712	30
	Block-Centric	0.94 s	1,747,653	6
Friendster	**Vertex-Centric**	120.24 s	7,226,963,186	22
	Block-Centric	2.52 s	19,410,865	5
USA Road	**Vertex-Centric**	510.98s	8,353,044,435	6,262
	Block-Centric	1.94 s	270,257	26

Figure 1: Overall Performance of Hash-Min

In addition, as long as the number of blocks is sufficiently larger than the number of workers, the block-centric model can achieve a rather balanced workload with a simple greedy algorithm for block-to-worker assignment described in Section 4.

Figure 1 reports the overall performance (elapsed computing time, total number of messages sent, and total number of supersteps) of the vertex-centric and block-centric Hash-Min algorithms on three graphs: an RDF graph *BTC*, a social network *Friendster*, and a *USA* road network. The details of these graphs can be found in Section 8. Figure 2 reports the performance (running time and number of messages) of Hash-Min for each superstep on *BTC* and *Friendster*.

BTC has skewed degree distribution, as a few vertices have degree over a million but the average degree is only 4.69. The vertex-centric program is severely affected by the skewed workload due to high-degree vertices in the first few supersteps, and a large number of messages are passed as shown in Figure 2(a). The number of messages starts to drop significantly only in superstep 4 when the few extremely high-degree vertices become inactive. The subsequent supersteps involve less and less messages since the smallest vertex IDs have been seen by most vertices, and hence only a small fraction of the vertices remain active. However, it still takes 30 supersteps to complete the job. On the contrary, our block-centric program has balanced workload from the beginning and uses only 6 supersteps. Overall, as Figure 1 shows, the block-centric program uses 680 times less messages and 30 times less computing time.

Next, we discuss the processing of graphs with high density. Social networks and mobile phone networks often have relatively higher average degree than other real-world graphs. For example, Friendster has average degree of 55.06 while the USA road network has average degree of only 2.44 (see Figure 7), which implies that a vertex of Friendster can send/receive 22.57 times more messages than that of the USA road network. The total number of messages in each superstep of the vertex-centric Hash-Min is bounded by $O(|E|)$, while that of the block-centric Hash-Min is bounded by $O(|\mathbb{E}|)$. Since $|\mathbb{E}|$ is generally significantly smaller than $|E|$, the block-centric model is much cheaper.

As Figure 1 shows, the vertex-centric Hash-Min uses 372 times more messages than the block-centric Hash-Min on Friendster, resulting in 48 times longer computing time. Figure 2(b) further shows that the number of messages and the elapsed computing time per superstep of the vertex-centric program is significantly larger than that of the block-centric program. Note that the more message passing and longer computing time are not primarily due to skewed workload because the maximum degree in Friendster is only 5214.

Let $\delta(G)$ and $\delta(\mathbb{G})$ be the diameter of G and $\mathbb{G}$. The vertex-centric Hash-Min takes $O(\delta(G))$ supersteps, while the block-centric Hash-Min takes only $O(\delta(\mathbb{G}))$ supersteps. For real-world graphs

Superstep #		1	2	3	4	5	6	7	8	9	...	29	30
Vertex-Centric	Time	7.24 s	6.82 s	6.62 s	2.86 s	2.34 s	0.17 s	0.13 s	0.10 s	0.09s	...	0.08 s	0.10 s
	Msg #	393,175,048	349,359,723	320,249,398	78,466,694	44,961,718	1,884,460	530,278	128,602	61,727		4	0
Block-Centric	Time	0.29 s	0.12 s	0.10 s	0.15 s	0.12 s	0.15 s						
	Msg #	1,394,408	294,582	55,775	2,848	40	0						

(a) Per-Superstep Performance over BTC

Superstep #		1	2	3	4	5	6	7	8	...	21	22
Vertex-Centric	Time	26.86 s	27.64 s	27.86 s	26.97 s	8.96 s	0.43 s	0.15 s	0.11 s	...	0.18 s	0.15 s
	Msg #	1,725,523,081	1,719,438,065	1,717,496,808	1,636,980,454	416,289,356	8,780,258	1,484,531	587,275		1	0
Block-Centric	Time	0.53 s	1.53 s	0.25 s	0.10 s	0.06 s						
	Msg #	6,893,957	6,892,723	5,620,051	4,134	0						

(b) Per-Superstep Performance over Friendster

Figure 2: Performance of Hash-Min Per Superstep

with a large diameter, such as road networks, condensing a large region (i.e., a block) of G into a single vertex in $\mathbb{G}$ allows distant points to be reached within a much smaller number of hops. For example, in the USA road network, there may be thousands of hops between Washington and Florida; but if we condense each state into a block, then the two states are just a few hops apart.

The adverse effect of large diameter on the vertex-centric model can be clearly demonstrated by the USA road network. As shown in Figure 1, the vertex-centric Hash-Min takes in 6,262 supersteps, while the block-centric Hash-Min runs on a block-level graph with a much small diameter and uses only 22 supersteps. The huge number of supersteps of the vertex-centric Hash-Min also results in 263 times longer computing time and 30,908 times more messages being passed than the block-centric algorithm.

4. SYSTEM OVERVIEW

We first give an overview of the Blogel framework. Blogel supports three types of jobs: (1)*vertex-centric graph computing*, where a worker is called a *V-worker*; (2)*graph partitioning* which groups vertices into blocks, where a worker is called a *partitioner*; (3)*block-centric graph computing*, where a worker is called a *B-worker*.

Figure 3 illustrates how the three types of workers operate. Figure 3(a) shows that each V-worker loads its own portion of vertices, performs vertex-centric graph computation, and then dumps the output of the processed vertices (marked as grey) to HDFS. Figure 3(b) shows block-centric graph computing, where we assume that every block contains two vertices. Specifically, the vertices are first grouped into blocks by the partitioners, which dump the constructed blocks to HDFS. These blocks are then loaded by the B-workers for block-centric computation.

Blocks and B-workers. In both vertex-centric graph computing and graph partitioning, when a vertex v sends a message to a neighbor $u \in \Gamma(v)$, the worker to which the message is to be sent is identified by hashing $id(u)$. We now consider block-centric graph computing. Suppose that the vertex-to-block assignment and block-to-worker assignment are already given, we define the block of a vertex v by $block(v)$ and the worker of a block B by $worker(B)$. We also define $worker(v) = worker(block(v))$. Then, the ID of a vertex v in a B-worker is now given by a triplet $trip(v) = \langle id(v), block(v), worker(v)\rangle$. Thus, a B-worker can obtain the worker and block of a vertex v by checking its ID $trip(v)$.

Similar to a vertex in Pregel, a block in Blogel also has a *compute*() function. We use *B-compute*() and *V-compute*() to denote the *compute*() function of a block and a vertex, respectively. A block has access to all its vertices, and can send messages to any block B or vertex v as long as $worker(B)$ or $worker(v)$ is available. Each B-worker maintains two message buffers, one for exchanging vertex-level messages and the other for exchanging block-level messages. A block also has a state indicating whether it is active, and may vote to halt.

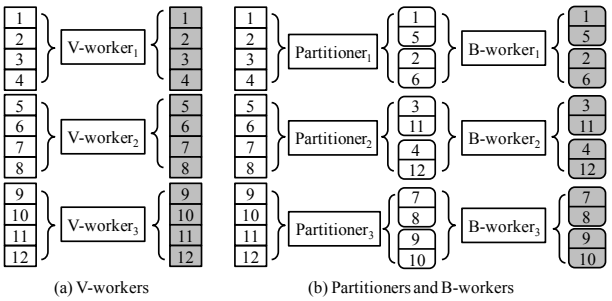


Figure 3: Operating Logic of Different Types of Workers

Blogel computing modes. Blogel operates in one of the following three computing modes, depending on the application:

- **B-mode.** In this mode, Blogel only calls *B-compute*() for all its blocks, and only block-level message exchanges are allowed. A job terminates when all blocks voted to halt and there is no pending message for the next superstep.
- **V-mode.** In this mode, Blogel only calls *V-compute*() for all its vertices, and only vertex-level message exchanges are allowed. A job terminates when all vertices voted to halt and there is no pending message for the next superstep.
- **VB-mode.** In this mode, in each superstep, a B-worker first calls *V-compute*() for all its vertices, and then calls *B-compute*() for all its blocks. If a vertex v receives a message at the beginning of a superstep, v is activated along with its block $B = block(v)$, and B will call its *B-compute*() function. A job terminates only if all vertices and blocks voted to halt and there is no pending message for the next superstep.

Partitioners. Blogel supports several types of pre-defined partitioners. Users may also implement their own partitioners in Blogel. A partitioner loads each vertex v together with $\Gamma(v)$. If the partitioning algorithm supports only undirected graphs but the input graph G is directed, partitioners first transform G into an undirected graph, by making each edge bi-directed. The partitioners then compute the vertex-to-block assignment (details in Section 7).

Block assignment. After graph partitioning, $block(v)$ is computed for each vertex v. The partitioners then compute the block-to-worker assignment. This is actually a load balancing problem:

DEFINITION 1 (LOAD BALANCING PROBLEM [9]). *Given k workers $w_1, \ldots, w_k$, and n jobs, let $J(i)$ be the set of jobs assigned to worker w_i and c_j be the cost of each job j. The load of worker w_i is defined as $L_i = \sum_{j \in J(i)} c_j$. The goal is to minimize $L = \max_i L_i$.*

In our setting, each job corresponds to a block B, whose cost is given by the number of vertices in B. We use the 4/3-approximation

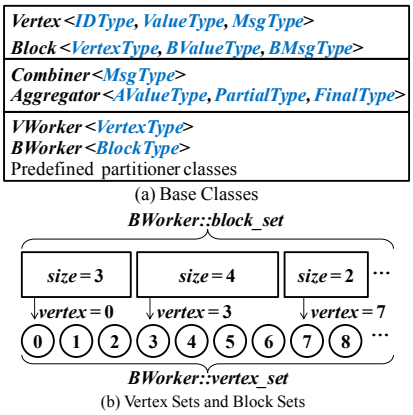


Figure 4: Programming Interface of Blogel

algorithm given in [5], which first sorts the jobs by cost in non-increasing order, and then scans through the sorted jobs and assigns each job to the worker with the smallest load.

The block-to-worker assignment is computed as follows. (1)To get the block sizes, each partitioner groups its vertices into blocks and sends the number of vertices in each block to the master. The master then aggregates the numbers sent by the partitioners to obtain the global number of vertices for each block. (2)The master then computes the block-to-worker assignment using the greedy algorithm described earlier, and broadcasts the assignment to each partitioner. (3)Each partitioner sets $worker(v)$ for each of its vertex v according to the received block-to-worker assignment (note that $block(v)$ is already computed).

Triplet ID of neighbors. So far we only compute $trip(v)$ for each vertex v. However, in block-centric computing, if a vertex v needs to send a message to a neighbor $u \in \Gamma(v)$, it needs to first obtain the worker holding u from $trip(u)$. Thus, we also compute $\widehat{\Gamma}(v) = \{trip(u) : u \in \Gamma(v)\}$ for each vertex v as follows. Each worker w_i constructs a look-up table $LT_{i\to j}$ locally for every worker w_j in the cluster: for each vertex v in w_i, and for each neighbor $u \in \Gamma(v)$, $trip(v)$ is added to $LT_{i\to j}$, where $j = hash(id(u))$, i.e., u is on worker w_j. Then, w_i sends $LT_{i\to j}$ to each w_j, and each worker merges the received look-up tables into one look-up table. Now, a vertex u on worker w_j can find $trip(v)$ for each neighbor $v \in \Gamma(u)$ from the look-up table of w_j to construct $\widehat{\Gamma}(u)$.

Till now, each partitioner w_i still holds only those vertices v with $hash(id(v)) = i$. After $\widehat{\Gamma}(v)$ is constructed for each vertex v, the partitioners exchange vertices according to the block-to-worker assignment. Each partitioner w_i then dumps its vertices to an HDFS file, which is later loaded by the corresponding B-worker w_i during block-centric computing. Each vertex v has an extra field, $content(v)$, that keeps additional information such as edge weight and edge direction during data loading. It is used along with $trip(v)$ and $\widehat{\Gamma}(v)$, to format v's output line during data dumping.

5. PROGRAMMING INTERFACE

Similar to Pregel, writing a Blogel program involves subclassing a group of predefined classes, with the template arguments of each base class properly specified. Figure 4(a) shows the base classes of Blogel. We illustrate their usage by showing how to implement the *Hash-Min* algorithm described in Section 3.

5.1 Vertex-Centric Interface

In the vertex-centric model, each vertex v maintains an integer ID $id(v)$, an integer field $cc(v)$ and a list of neighbors $\Gamma(v)$. Thus, in the *Vertex* class, we specify *IDType* as integer, *ValueType* as a user-defined type for holding both $cc(v)$ and $\Gamma(v)$, and *MsgType* as integer since *Hash-Min* sends messages in the form of vertex ID. We then define a class *CCVertex* to inherit this class, and implement the *compute*() function using the *Hash-Min* algorithm in Section 3. The *Vertex* class also has another template argument (not shown in Figure 4) for specifying the vertex-to-worker assignment function over *IDType*, for which a hash function is specified by default.

To run the *Hash-Min* job, we inherit the *VWorker*<*CCVertex*> class, and implement two functions: (1)*load_vertex*(*line*), which specifies how to parse a line from the input file into a vertex object; (2)*dump_vertex*(*vertex*, *HDFSWriter*), which specifies how to dump a vertex object to the output file. *VWorker*'s *run*() function is then called to start the job. *BWorker* and the partitioner classes also have these three functions, though the logic of *run*() is different.

5.2 Block-Centric Interface

For the block-centric model, in the *Vertex* class, we specify *IDType* as a triplet for holding $trip(v)$, *ValueType* as a list for holding $\widehat{\Gamma}(v)$, while *MsgType* can be any type since the algorithm works in B-mode and there is no vertex-level message exchange. We then define a class *CCVertex* that inherits this class with an empty *V-compute*() function. In the *Block* class, we specify *VertexType* as *CCVertex*, *BValueType* as a block-level adjacency list (i.e., $\Gamma(B)$), and *BMsgType* as integer since the algorithm sends messages in the form of block ID. We then define a class *CCBlock* that inherits this *Block* class, and implement the *B-compute*() function using the logic of block-centric *Hash-Min*. Finally, we inherits the *BWorker*<*CCBlock*> class, implement the vertex loading/dumping functions, and call *run*() to start the job.

A *Block* object also has the following fields: (1)an integer block ID, (2)an array of the block's vertices, denoted by $vertex$, (3)and the number of vertices in the block, denoted by $size$.

A *BWorker* object also contains an array of blocks, denoted by $block\_set$, and an array of vertices, denoted by $vertex\_set$. As shown in Figure 4(b), the vertices in a B-worker's $vertex\_set$ are grouped by blocks; and for each block B in the B-worker's $block\_set$, B's $vertex$ field is actually a pointer to the first vertex of B's group in the B-worker's $vertex\_set$. Thus, a block can access its vertices in *B-compute*() as $vertex[0]$, $vertex[1]$, ..., $vertex[size-1]$.

A subclass of *BWorker* also needs to implement an additional function *block_init*(), which specifies how to set the user-specified field *BValueType* for each block. After a B-worker loads all its vertices into *vertex_set*, it scans *vertex_set* to construct its *block_set*, where the fields *vertex* and *size* of each block are automatically set. Then, before the block-centric computing begins, *block_init*() is called to set the user-specified block field. For our block-centric *Hash-Min* algorithm, in *block_init*(), each block B constructs $\Gamma(B)$ from $\Gamma(v)$ of all $v \in V(B)$.

5.3 Global Interface

In Blogel, each worker is a computer process. We now look at a number of fields and functions that are global to each process. They can be accessed in both *V-compute*() and *B-compute*().

Message buffers and combiners. Each worker maintains two message buffers, one for exchanging vertex-level messages and the other for exchanging block-level messages. A combiner is associated with each message buffer, and it is not defined by default. To use a combiner, we inherit the *Combiner* class and implement the *combine*() function, which specifies the combiner logic. When a vertex or a block calls the *send_msg*(*target*, *msg*) function of the message buffer, *combine*() is called if the combiner is defined.

Aggregator. An aggregator works as follows. In each superstep, a vertex/block's *V-compute*()/*B-compute*() function may call *aggregate*(*value*), where *value* is of type *AValueType*. After a worker calls *V-compute*()/*B-compute*() for all vertices/blocks, the aggregated object maintained by its local aggregator (of type *PartialType*) is sent to the master. When the master obtains the locally aggregated objects from all workers, it calls *master_compute*() to aggregate them to a global object of type *FinalType*. This global object is then broadcast to all the workers so that it is available to every vertex/block in the next superstep.

To define an aggregator, we subclass the *Aggregator* class to include a field recording the aggregated values, denoted by agg, and implement *aggregate*(*value*) and *master_compute*(). An object of this class is then assigned to the worker.

Blogel also supports other useful global functions such as graph mutation (which are user-defined functions to add/delete vertices/edges), and the *terminate*() function which can be called to terminate the job immediately. In addition, Blogel maintains the following global fields which are useful for implementing the computing logic: (1)the ID of the current superstep, which also indicates the number of supersteps executed so far; (2)the total number of vertices among all workers at the beginning of the current superstep, whose value may change due to graph mutation; (3)the total number of active vertices among all workers at the beginning of the current superstep.

6. APPLICATIONS

We apply Blogel to solve four classic graph problems: *Connected Components (CCs)*, *Single-Source Shortest Path (SSSP)*, *Reachability*, and *PageRank*. In Sections 3 and 5, we have discussed how Blogel computes CCs with the *Hash-Min* logic in both the vertex-centric model, and B-mode of the block-centric model. We now discuss the solutions to the other three problems in Blogel.

6.1 Single-Source Shortest Path

Given a graph $G = (V, E)$, where each edge $(u, v) \in E$ has length $\ell(u, v)$, and a source $s \in V$, SSSP computes a shortest path from s to every other vertex $v \in V$, denoted by $SP(s, v)$.

Vertex-centric algorithm. We first discuss the vertex-centric algorithm, which is similar to Pregel's SSSP algorithm [11]. Each vertex v has two fields: $\langle prev(v), dist(v)\rangle$ and $\Gamma_{out}(v)$, where $prev(v)$ is the vertex preceding v on $SP(s, v)$ and $dist(v)$ is the length of $SP(s, v)$. Each $u \in \Gamma_{out}(v)$ is associated with $\ell(v, u)$.

Initially, only s is active with $dist(s)$=0, while $dist(v)$=∞ for any other vertex v. In superstep 1, s sends a message $\langle s, dist(s) + \ell(s, u)\rangle$ to each $u \in \Gamma_{out}(s)$, and votes to halt. In superstep i (i>1), if a vertex v receives messages $\langle w, d(w)\rangle$ from any of v's in-neighbor w, then v finds the in-neighbor w^* such that $d(w^*)$ is the smallest among all $d(w)$ received. If $d(w^*) < dist(v)$, v updates $\langle prev(v), dist(v)\rangle$=$\langle w^*, d(w^*)\rangle$, and sends a message $\langle v, dist(v)+\ell(v, u)\rangle$ to each out-neighbor $u \in \Gamma_{out}(v)$. Finally, v votes to halt.

Let $hop(s, v)$ be the number of hops of $SP(s, v)$, and $\mathcal{L} = \max_{v \in V} hop(s, v)$. The vertex-centric algorithm runs for $O(\mathcal{L})$ supersteps and in each superstep, at most one message is sent along each edge. Thus, the total workload is $O(\mathcal{L}(|V| + |E|))$.

Block-centric algorithm. Our block-centric solution operates in VB-mode. Each vertex maintains the same fields as in the vertex-centric algorithm, and blocks do not maintain any information. In each superstep, *V-compute*() is first executed for all vertices, where a vertex v finds w^* from the incoming messages as in the vertex-centric algorithm. However, now v votes to halt only if $d(w^*) \geq dist(v)$. Otherwise, v updates $\langle prev(v), dist(v)\rangle = \langle w^*, d(w^*)\rangle$ but stays active. Then, *B-compute*() is executed, where each block B collects all its active vertices v into a priority queue Q (with $dist(v)$ as the key), and makes these vertices vote to halt. *B-compute*() then runs Dijkstra's algorithm on B using Q, which removes the vertex $v \in Q$ with the smallest value of $dist(v)$ from Q for processing each time. The out-neighbors $u \in \Gamma(v)$ are updated as follows. For each $u \in V(B)$, if $dist(v)+\ell(v, u) < dist(u)$, we update $\langle prev(u), dist(u)\rangle$ to be $\langle v, dist(v)+\ell(v, u)\rangle$, and insert u into Q with key $dist(u)$ if $u \notin Q$, or update $dist(u)$ if u is already in Q. For each $u \notin V(B)$, a message $\langle v, dist(v) + \ell(v, u)\rangle$ is sent to u. B votes to halt when Q becomes empty. In the next superstep, if a vertex u receives a message, u is activated along with its block, and the block-centric computation repeats.

Compared with the vertex-centric algorithm, this algorithm saves a significant amount of communication cost since there is no message passing among vertices within each block. In addition, messages propagate from s in the unit of blocks, and thus, the algorithm requires much less than supersteps than $O(\mathcal{L})$.

For both the vertex-centric and block-centric algorithms, we apply a combiner as follows. Given a set of messages from a worker, $\{\langle w_1, d(w_1)\rangle, \langle w_2, d(w_2)\rangle, \ldots, \langle w_k, d(w_k)\rangle\}$, to be sent to a vertex u, the combiner combines them into a single message $\langle w^*, d(w^*)\rangle$ such that $d(w^*)$ is the smallest among all $d(w_i)$ for $1 \leq i \leq k$.

6.2 Reachability

Given a directed graph $G = (V, E)$, a source vertex s and a destination vertex t, the problem is to decide whether there is a directed path from s to t in G. We can perform a bidirectional breadth-first search (BFS) from s and t, and check whether the two BFSs meet at some vertex. We assign each vertex v a 2-bit field $tag(v)$, where the first bit indicates whether s can reach v and the second bit indicates whether v can reach t.

Vertex-centric algorithm. We first set $tag(s) = 10$, $tag(t) = 01$; and for any other vertex v, $tag(v) = 00$. Only s and t are active initially. In superstep 1, s sends its tag 10 to all $v \in \Gamma_{out}(s)$, and t sends its tag 01 to all $v \in \Gamma_{in}(t)$. They then vote to halt. In superstep i ($i > 1$), a vertex v computes the bitwise-OR of all messages it receives, which results in tag^*. If $tag^* = 11$, or if the bitwise-OR of tag^* and $tag(v)$ is 11, v sets $tag(v) = 11$ and calls *terminate*() to end the program since the two BFSs now meet at v; otherwise, if $tag(v) = 00$, v sets $tag(v) = tag^*$ and, either sends tag^* to all $u \in \Gamma_{out}(v)$ if $tag^* = 10$, or sends tag^* to all $u \in \Gamma_{in}(v)$ if $tag^* = 01$. Finally, v votes to halt.

Note that if we set t to be a non-existent vertex ID (e.g., -1), the algorithm becomes BFS from s. We now analyze the cost of doing BFS. Let V_h be the set of vertices that are h hops away from s. We can prove by induction that, in superstep i, only those vertices in V_i both receive messages (from the vertices in V_{i-1}) and send messages to their out-neighbors. If a vertex in V_j ($j < i$) receives a message, it simply votes to halt without sending messages; while all vertices in V_j ($j > i$) remain inactive as they are not reached yet. Thus, the total workload is $O(|E| + |V|)$.

Block-centric algorithm. This algorithm operates in VB-mode. In each superstep, *V-compute*() is first called where each vertex v receives messages and updates $tag(v)$ as in the vertex-centric algorithm. If $tag(v)$ is updated, v remains active; otherwise, v votes to halt. Then, *B-compute*() is called, where each block B collects all its active vertices with tag 10 (01) into a queue Q_s (Q_t). If an active vertex with tag 11 is found, B calls *terminate*(). Otherwise, B performs a forward BFS using Q_s as follows. A vertex v is dequeued from Q_s each time, and the out-neighbors $u \in \Gamma_{out}(v)$ are updated as follows. For each out-neighbor $u \in V(B)$, if $tag(u) = 00$, $tag(u)$ is set to 10 and u is enqueued; if $tag(u) = 01$ or 11, $tag(u)$ is set to 11 and *terminate*() is called. For each out-neighbor

	$\|V\|$	$\|E\|$	Kendall Tau Distance
BerkStan	685,230	7,600,595	834,804,094
Google	875,713	5,105,039	2,185,214,827
NotreDame	325,729	1,497,134	1,008,151,095
Stanford	281,903	2,312,497	486,171,631

Figure 5: Impact of PageRank Loss

$u \notin V(B)$, a message 10 is sent to u. Then a backward BFS using Q_t is performed in a similar way. Finally, B votes to halt. In the next superstep, if a vertex u receives a message, u is activated along with its block, and the block-centric computation repeats.

6.3 PageRank

Given a directed graph $G = (V, E)$, the problem is to compute the PageRank of each vertex $v \in V$. Let $pr(v)$ be the PageRank of v. Pregel's PageRank algorithm [11] works as follows. In superstep 1, each vertex v initializes $pr(v) = 1/|V|$ and distributes $pr(v)$ to the out-neighbors by sending each one $pr(v)/|\Gamma_{out}(v)|$. In superstep i ($i > 1$), each vertex v sums up the received PageRank values, denoted by sum, and computes $pr(v) = 0.15/|V| + 0.85 \times sum$. It then distributes $pr(v)/|\Gamma_{out}(v)|$ to each out-neighbor. A combiner is used, which aggregates the messages to be sent to the same destination vertex into a single message that equals their sum.

PageRank loss. Conceptually, the total amount of PageRank values remain to be 1, with 15% held evenly by the vertices, and 85% redistributed among the vertices by propagating along the edges. However, if there exists a sink page v (i.e., $\Gamma_{out}(v) = \emptyset$), $pr(v)$ is not distributed to any other vertex in the next superstep and the value simply gets lost. Therefore, in [11]'s PageRank algorithm, the total amount of PageRank values decreases as the number of supersteps increases.

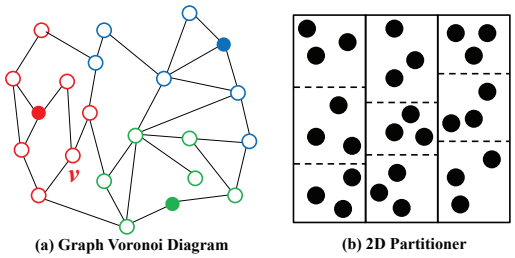
Let $V_0 = \{v \in V : \Gamma_{out}(v) = \emptyset\}$ be the set of *sink vertices*, i.e., vertices with out-degree 0. We ran [11]'s PageRank algorithm on two web graphs (listed in Figure 7) and computed the PageRank loss: (1)*WebUK* which has 9.08% of the vertices being sink vertices, and (2)*WebBase* which has 23.41% of the vertices being sink vertices. We found that the algorithm converges in 88 supersteps with 17% PageRank loss on *WebUK*, and in 79 supersteps with 34% PageRank loss on *WebBase*. As we shall see shortly, such a large PageRank loss reveals a problem that must be addressed.

Vertex-centric algorithm. A common solution to the PageRank loss problem is to make each sink vertex $v \in V_0$ link to all the vertices in the graph[1], i.e. to distribute $pr(v)/|V|$ to each vertex. Intuitively, this models the behavior of a random surfer: if the surfer arrives at a sink page, it picks another URL at random and continues surfing again. Since $|V|$ is usually large, $pr(v)/|V|$ is small and the impact of v to the PageRank of other vertices is negligible.

Compared with the standard PageRank definition above, Pregel's PageRank algorithm [11] changes the relative importance order of the vertices. To illustrate, we compute PageRank on the four web graphs from the SNAP database[2] using the algorithm in [11] and the standard PageRank definition, and compare the ordered vertex lists obtained. The results in Figure 5 show that the two vertex lists have a large Kendall tau distance. For example, the graph *Google* has over 2 million vertex pairs whose order of PageRank magnitude is reversed from the standard definition.

Obviously, materializing $\Gamma_{out}(v) = V$ for each $v \in V_0$ is unacceptable both in space and in communication cost. We propose an aggregator-based solution. In *compute*(), if v's out-degree is 0, v provides $pr(v)$ to an aggregator that computes $agg = \sum_{v \in V_0} pr(v)$.

[1]http://www.google.com/patents/US20080075014
[2]http://snap.stanford.edu/data/



(a) Graph Voronoi Diagram (b) 2D Partitioner

Figure 6: Partitioners (Best Viewed in Colors)

The PageRank of v is now updated by $pr(v) = 0.15/|V| + 0.85 \times (sum + agg/|V|)$, where sum is compensated with $agg/|V| = \sum_{v \in V_0} \frac{pr(v)}{|V|}$.

Let $pr_i(v)$ be the PageRank of v in superstep i. Then, PageRank computation stops if $|pr_i(v) - pr_{i-1}(v)| < \epsilon/|V|$ for all $v \in V$ (note that the average PageRank is $1/|V|$). We set ϵ to be 0.01 throughout this paper.

We also implement this stop condition using an aggregator that performs logic AND. In *compute*(), each vertex v provides '*true*' to the aggregator if $|pr_i(v) - pr_{i-1}(v)| < \epsilon/|V|$, and '*false*' otherwise. Moreover, if v finds that the aggregated value of the previous superstep is '*true*', it votes to halt directly without doing PageRank computation.

Block-centric algorithm. In a web graph, each vertex is a web page with a URL (e.g., `cs.stanford.edu/admissions`), and we can naturally group all vertices with the same host name (e.g., `cs.stanford.edu`) into a block. Kamvar et al. [6] proposed to initialize the PageRank values by exploiting this block structure, so that PageRank computation can converge faster. Note that though a different initialization is used, the PageRank values still converge to a unique stable state according to the Perron-Frobenius theorem.

The implementation of the algorithm of [6] in the Blogel framework consists of two jobs. The first job operates in B-mode. Before computation, in *block_init*(), each block B first computes the local PageRank of each $v \in V(B)$, denoted by $lpr(v)$, by a single-machine PageRank algorithm with B as input. Block B then constructs $\Gamma(B)$ from $\Gamma(v)$ of all $v \in V(B)$ using [6]'s approach, which assigns a weight to each out-edge. Finally, *B-compute*() computes the BlockRank of each block $B \in \mathbb{B}$, denoted by $br(B)$, on $\mathbb{G} = (\mathbb{B}, \mathbb{E})$ using a logic similar to the vertex-centric PageRank, except that BlockRank is distributed to out-neighbors proportionally to the edge weights. The second job operates in V-mode, which initializes $pr(v) = lpr(v) \cdot br(block(v))$ [6], and performs the standard PageRank computation on G.

7. PARTITIONERS

Efficient computation of blocks that give balanced workload is crucial to the performance of Blogel. We have discussed the logic of partitioners such as the computation of the block-to-worker assignment in Section 4. We have also seen a URL-based partitioner for web graphs in Section 6.3, where the vertex-to-block assignment is determined by the host names extracted from URLs. In this section, we introduce two other types of partitioners, with an emphasis on computing the vertex-to-block assignment.

7.1 Graph Voronoi Diagram Partitioner

We first review the *Graph Voronoi Diagram* (GVD) [3] of an undirected unweighted graph $G = (V, E)$. Given a set of source vertices $s_1, s_2, \ldots, s_k \in V$, we define a partition of V: $\{VC(s_1), VC(s_2), \ldots, VC(s_k)\}$, where a vertex v is in $VC(s_i)$ only if s_i is closer to v (in terms of the number of hops) than any other source.

Ties are broken arbitrarily. The set $VC(s_i)$ is called the Voronoi cell of s_i, and the Voronoi cells of all sources form the GVD of G.

Figure 6(a) illustrates the concept of GVD, where source vertices are marked with solid circles. Consider vertex v in Figure 6(a), it is at least 2, 3 and 5 hops from the red, green and blue sources. Since the red source is closer to v, v is assigned to the Voronoi cell of the red source. All the vertices in Figure 6(a) are partitioned into three Voronoi cells, where the vertices with the same color belong to the same Voronoi cell.

The GVD computation can be easily implemented in the vertex-centric computing model, by performing multi-source BFS. Specifically, in superstep 1, each source s sets $block(s) = s$ and broadcasts it to the neighbors; for each non-source vertex v, $block(v)$ is unassigned. Finally, the vertex votes to halt. In superstep i ($i > 1$), if $block(v)$ is unassigned, v sets $block(v)$ to an arbitrary source received, and broadcasts $block(v)$ to its neighbors before voting to halt. Otherwise, v votes to halt directly. When the process converges, we have $block(v) = s_i$ for each $v \in VC(s_i)$.

The multi-source BFS has linear workload since each vertex only broadcasts a message to its neighbors when $block(v)$ is assigned, and thus the total messages exchanged by all vertices is bounded by $O(|E|)$. However, we may obtain some huge Voronoi cells (or blocks), which are undesirable for load balancing. We remark that block sizes can be aggregated at the master in a similar manner as when we compute the block-to-worker assignment in Section 4.

Our GVD partitioner works as follows, where we use a parameter $\underline{b_{max}}$ to limit the maximum block size. Initially, each vertex v samples itself as a source with probability $\underline{p_{samp}}$. Then, multi-source BFS is performed to partition the vertices into blocks. If the size of a block is larger than b_{max}, we set $block(v)$ unassigned for any vertex v in the block (and reactivate v). We then perform another round of source sampling and multi-source BFS on those vertices v with $block(v)$ unassigned, using a higher sampling probability. Here, we increase p_{samp} by a factor of $\underline{f}$ ($f \geq 1$) after each round in order to decrease the chance of obtaining an over-sized block. This process is repeated until the stop condition is met.

We check two stop conditions, and the process described above stops as long as one condition is met: (1)let $A(i)$ be the number of active vertices at the beginning of the i-th round of multi-source BFS, then the process stops if $A(i)/A(i-1) > \gamma$. Here, $\underline{\gamma} \leq 1$ is a parameter determining when multi-source BFS is no longer effective in assigning blocks; or (2)the process stops if $p_{samp} > p_{max}$, where $\underline{p_{max}}$ is the maximum allowed sampling rate (recall that $p_{samp} = f \cdot p_{samp}$ after each round).

Moreover, to prevent multi-source BFS from running too many supersteps, which may happen if graph diameter δ is large and the sampling rate p_{samp} is small, we include a user-specified parameter $\underline{\delta_{max}}$ to bound the maximum number of supersteps, i.e., each vertex votes to halt in superstep δ_{max} during multi-source BFS.

When the above process terminates, there may still be some vertices not assigned to any block. This could happen since multi-source BFS works well on large CCs, as a larger CC tends to contain more sampled sources, but the sampling is ineffective for handling small CCs. For example, consider the extreme case where the graph is composed of isolated vertices. Since each round of multi-source BFS assigns block ID to only around $p_{samp}|V|$ vertices, it takes around $1/p_{samp}$ (which is 1000 if $p_{samp} = 0.1\%$) rounds to assign block ID to all vertices, which is inefficient.

Our solution to assigning blocks for small CCs is by the *Hash-Min* algorithm, which marks each small CC as a block using only a small number of supersteps. Specifically, after the rounds of multi-source BFS terminate, if there still exists any unassigned vertex, we run Hash-Min on the subgraph induced by these unassigned vertices. We call this step as *subgraph Hash-Min*.

There are six parameters, (p_{samp}, δ_{max}, b_{max}, f, γ, p_{max}), that need to be specified for the partitioner. We show that these parameters are intuitively easy to set. In particular, we found that the following setting of the parameters work well for most large real-world graphs. The sampling rate p_{samp} decides the number of blocks, and usually a small value as 0.1% is a good choice. Note that p_{samp} cannot be too small in order not to create very large blocks. As for the stopping parameters, γ is usually set as 90%, and p_{max} as 10% with $f = 2$, so that there will not be too many rounds of multi-source BFS. The bound on the number of supersteps, δ_{max}, is set to be a tolerable number such as 50, but for small-diameter graphs (e.g., social networks) we can set a smaller δ_{max} such as 10 since the number of supersteps needed for such graphs is small. We set b_{max} to be 100 times that of the expected block size (e.g., $b_{max} = 100,000$ when $p_{samp} = 0.1\%$) for most graphs, except that for spatial networks the block size is limited by δ_{max} already and hence we just set it to ∞. We will further demonstrate in our experiments that the above settings work effectively for different types of real-world graphs.

7.2 2D Partitioner

In many spatial networks, vertices are associated with (x, y)-coordinates. Blogel provides a 2D partitioner to partition such graphs. A 2D partitioner associates each vertex v with an additional field (x, y), and it consists of two jobs.

The first job is vertex-centric and works as follows: (1)each worker samples a subset of its vertices with probability $\underline{p_{samp}}$ and sends the sample to the master; (2)the master first partitions the sampled vertices into $\underline{n_x}$ slots by the x-coordinates, and then each slot is further partitioned into n_y slots by the y-coordinates. Each resulting slot is a *super-block*. Figure 6(b) shows a 2D partitioning with n_x=n_y=3. This partitioning is broadcast to the workers, and each worker assigns each of its vertices to a super-block according to the vertex coordinates. Finally, vertices are exchanged according to the superblock-to-worker assignment computed by master, with $\widehat{\Gamma}(v)$ constructed for each vertex v as described in Section 4.

Since a super-block may not be connected, we perform a second block-centric job, where workers run BFS over their super-blocks to break them into connected blocks. Each worker marks the blocks it obtains with IDs 0, 1, $\cdots$. To get the global block ID, each worker w_i sends the number of blocks it has, denoted by $|\mathbb{B}_i|$, to the master which computes for each worker w_j a prefix sum $sum_j = \sum_{i<j} |\mathbb{B}_i|$, and sends sum_j to w_j. Each worker w_j then adds sum_j to the block IDs of its blocks, and hence each block obtains a unique block ID. Finally, $\widehat{\Gamma}(v)$ are updated for each vertex v.

The 2D partitioner has parameters (p_{samp}, n_x, n_y). The setting of p_{samp} is similar to the GVD partitioner. To set n_x and n_y, we use $\delta(\mathbb{G}) \approx O(\sqrt{\max\{n_x, n_y\}})$ as a guideline, since the diameter of $\mathbb{G}$ is critical to the performance of block-centric algorithms.

8. EXPERIMENTS

We compare the performance of Blogel and with Giraph 1.0.0, Giraph++[3] and GraphLab 2.2 (which includes the features of PowerGraph [4]). We ran our experiments on a cluster of 16 machines, each with 24 processors (two Intel Xeon E5-2620 CPU) and 48GB RAM. One machine is used as the master that runs only one worker, while the other 15 machines act as slaves running multiple workers. The connectivity between any pair of nodes in the cluster is 1Gbps.

[3] https://issues.apache.org/jira/browse/GIRAPH-818

	Data	Type	|V|	|E|	AVG Deg	Max Deg
Web Graphs	WebUK	directed	133,633,040	5,507,679,822	41.21	22,429
	WebBase	directed	118,142,155	1,019,903,190	8.63	3,841
Social Networks	Friendster	undirected	65,608,366	3,612,134,270	55.06	5,214
	LiveJournal	directed	10,690,276	224,614,770	21.01	1,053,676
RDF	BTC	undirected	164,732,473	772,822,094	4.69	1,637,619
Spatial Networks	USA Road	undirected	23,947,347	58,333,344	2.44	9
	Euro Road	undirected	18,029,721	44,826,904	2.49	12

Figure 7: Datasets

For the experiments of Giraph, we use the multi-threading feature added by Facebook, and thus each worker refers to a computing thread. However, Giraph++ is built on top of an earlier Giraph version by Yahoo!, which does not support multi-threading. We therefore run multiple workers (mapper tasks) per machine. We make all Giraph++ codes used in our experiments public[4].

We used seven large real-world datasets, which are from four different domains as shown in Figure 7: (1)web graphs: *WebUK*[5] and *WebBase*[6]; (2)social networks: *Friendster*[7] and *LiveJournal*[8]; (3)RDF graph: *BTC*[9] (a graph converted from the Billion Triple Challenge 2009 RDF dataset [2]); (4)road networks: *USA* and *Euro*[10].

Among them, *WebUK*, *LiveJournal* and *BTC* have skewed degree distribution; *WebUK*, *Friendster* and *LiveJournal* have average degree relatively higher than other large real-world graphs; *USA* and *Euro*, as well as *WebUK*, have a large diameter.

For a graph of certain size, we need a certain amount of computing resources (i.e., workers) to achieve good performance. However, the performance does not further improve if we increase the number of workers per slave machine beyond that amount, since the increased overhead of inter-machine communication outweighs the increased computing power. In the experiments, we run 10 workers for *WebUK* and *WebBase*, 8 for *Friendster*, 2 for *LiveJournal*, 4 for *BTC*, *USA* and *Euro*, which exhibit good performance.

8.1 Blogel Implementation

We make Blogel open-source. All the system source codes, as well as the source codes of the applications discussed in this paper, can be found in `http://www.cse.cuhk.edu.hk/blogel`.

Blogel is implemented in C++ as a group of header files, and users only need to include the necessary base classes and implement the application logic in their subclasses. Blogel communicates with HDFS through libhdfs, a JNI based C API for HDFS. Each worker is simply an MPI process and communications are implemented using MPI communication primitives. While one may deploy Blogel with any Hadoop and MPI version, we use Hadoop 1.2.1 and MPICH 3.0.4 in our experiments. All programs are compiled using GCC 4.4.7 with -O2 option enabled.

Blogel makes the master a worker, and fault recovery can be implemented by a script as follows. The script loops a Blogel job, which runs for at most Δ supersteps before dumping the intermediate results to HDFS. Meanwhile, the script also monitors the cluster condition. If a machine is down, the script kills the current job and restarts another job loading the latest intermediate results from HDFS. Fault tolerance is achieved by the data replication in HDFS.

8.2 Performance of Partitioners

We first report the performance of Blogel's partitioners.

[4] https://issues.apache.org/jira/browse/GIRAPH-902

[5] http://law.di.unimi.it/webdata/uk-union-2006-06-2007-05

[6] http://law.di.unimi.it/webdata/webbase-2001

[7] http://snap.stanford.edu/data/com-Friendster.html

[8] http://konect.uni-koblenz.de/networks/livejournal-groupmemberships

[9] http://km.aifb.kit.edu/projects/btc-2009/

[10] http://www.dis.uniroma1.it/challenge9/download.shtml

Performance of GVD partitioners. Figure 8 shows the performance of the GVD partitioners, along with the parameters we used. The web graphs can be partitioned simply based on URLs, but we also apply GVD partitioners to them for the purpose of comparison.

The partitioning parameters are set according to the heuristics given in Section 7, but if there exists a giant block in the result which is usually obtained in the phase when Hash-Min is run, we check why the multi-source BFS phase terminates. If it terminates because the sampling rate increases beyond p_{max}, we decrease f and increase p_{max} and γ to allow more rounds of multi-source BFS to be run in order to break the giant block into smaller ones. We also increase b_{max} to relax the maximum block size constraint during multi-source BFS, which may generate some relatively larger blocks (still bounded by b_{max}) but reduce the chance of producing a giant block (not bounded by b_{max}). We slightly adjusted the parameters for *WebUK*, *WebBase* and *BTC* in this way, while all other datasets work well with the default setting described in Section 7.

Recall from Section 4 that besides block-to-worker assignment, partitioners also compute vertex-to-block assignment (denoted by "Block Assign." in Figure 8), construct $\widehat{\Gamma}(v)$ (denoted by "Triplet ID Neighbors"), and exchange vertices according to the block-to-worker assignment (denoted by "Vertex Exchange"). We report the computing time for "Block Assign.", "Triplet ID Neighbors", and "Vertex Exchange" in Figure 8. We also report the data loading/dumping time and the total computation time of the GVD partitioners in the gray columns. As for vertex-to-block assignment, (1)for multi-source BFS, we report the number of rounds, the total number of supersteps taken by all these rounds, and the average time of a superstep; (2)for running *Hash-Min* on the subgraphs, we report the number of supersteps and the average time of a superstep.

As Figure 8 shows, the partitioning is very efficient as the computing time is comparable to graph loading/dumping time. In fact, within the overall computing time, a significant amount of time is spent on constructing $\widehat{\Gamma}(v)$ and vertex exchange, and the vertex-to-block assignment computation is highly efficient.

Figure 9 shows the number of blocks and vertices assigned to each worker, for example, *WebUK* is partitioned over workers 0 to 150 and each worker contains x blocks and y vertices, where $16,781 \leq x \leq 16,791$ and $884,987 \leq y \leq 884,988$. Thus, we can see that the GVD partitioners achieves very balanced block-to-worker assignment, and it also shows that the greedy algorithm described in Section 4 is effective. The workload is relatively less balanced only for *BTC*, where we have 13 larger blocks distributed over workers 0 to 12. This is mainly caused by the few vertices in *BTC* with very high degree (the max degree is 1.6M). We remark that probably no general-purpose partitioning algorithm is effective on *BTC*, if there exists a balanced partitioning for *BTC* at all.

Performance of 2D partitioners. Since the vertices in *USA* and *Euro* have 2D coordinates, we also run 2D partitioners on them, with $(p_{samp}, n_x, n_y) = (1\%, 20, 20)$. Figure 10(a) shows the quality of 2D partitioning, which reports the number of super-blocks, blocks and vertices in each worker. The number of blocks/vertices per worker obtained by 2D partitioning is not as even as that by GVD partitioning. However, the shape of the super-blocks are very regular, resulting in fewer number of edges crossing super-blocks and a smaller diameter for the block-level graph, and thus block-centric algorithms can run faster.

Figures 10(b) and 10(c) further show that 2D partitioning is also more efficient than GVD partitioning. For example, for *USA*, job 1 of 2D partitioning takes 26.92 seconds (58% spent on loading and dumping) and job 2 takes 13.29 seconds (93.5% spent on loading and dumping), which is still much shorter than the time taken by the GVD partitioner ($9.06 + 38.56 + 7.77 = 55.39$ seconds).

Data	Parameters						Performance										
							Load	Computation									Dump
								Multi-Source BFS			Subgraph Hash-Min		Block Assign.	Triplet ID Neighbors	Vertex Exchange	Total	
	p_{samp}	δ_{max}	b_{max}	f	γ	p_{max}		Round #	Step #	AVG	Step #	AVG					
WebUK	0.1%	30	500,000	1.6	100%	20%	135.28 s	12	291	0.71 s	8	0.22 s	8.31 s	111.00 s	200.62 s	714.68 s	403.63 s
WebBase	0.1%	30	500,000	2	100%	10%	36.20 s	6	180	0.50 s	70	0.16 s	9.24 s	40.49 s	43.76 s	248.31 s	50.43 s
Friendster	0.1%	10	100,000	2	90%	10%	71.89 s	2	13	4.14 s	12	0.14 s	2.38 s	55.09 s	70.45 s	204.36 s	223.93 s
LiveJournal	0.1%	10	100,000	2	90%	10%	35.08 s	7	64	0.23 s	9	0.20 s	0.87 s	6.14 s	8.56 s	36.19 s	13.71 s
BTC	0.1%	20	500,000	2	95%	10%	29.08 s	3	60	0.60 s	31	0.80 s	2.47 s	12.52 s	25.92 s	112.53 s	39.54 s
USA Road	0.1%	50	∞	2	90%	10%	9.06 s	4	166	0.07 s	19	0.04 s	3.08 s	3.35 s	6.23 s	38.56 s	7.77 s
Euro Road	0.1%	50	∞	2	90%	10%	7.17 s	4	171	0.07 s	17	0.04 s	1.90 s	2.78 s	4.88 s	30.96 s	5.84 s

Figure 8: Performance of Graph Voronoi Diagram Partitioners

	Super-Block #	Block #	Vertex #
USA	6 – 7	247 – 606	374,440 – 409,255
Euro	6 – 7	466 – 872	283,695 – 304,775

(a) Per-Worker Statistics

	Load	Partitioning					Dump
		Compute Slots	Block Assign.	Triplet ID Neighbors	Vertex Exchange	Total	
USA	7.49 s	1.75 s	0.03 s	3.70 s	5.82 s	11.31 s	8.12 s
Euro	6.02 s	1.43 s	0.03 s	3.21 s	4.73 s	9.40 s	5.69 s

(b) Job 1 Performance

	Load	Partitioning			Dump
		Compute Blocks	Triplet ID Neighbors	Total	
USA	1.37 s	0.50 s	0.37 s	0.87 s	11.05 s
Euro	1.08 s	0.30 s	0.38 s	0.69 s	5.98 s

(c) Job 2 Performance

Figure 10: Performance of 2D Partitioners

	Worker	Block #	Vertex #
WebUK	0 – 150	16,781 – 16,791	884,987 – 884,988
WebBase	0 – 150	19,329 – 19,341	782,398 – 782,399
Friendster	0 – 120	614 – 618	542,217 – 542,218
LiveJournal	0	5,066	344,848
	1 – 30	5,181 – 5,182	344,847 – 344,848
BTC	0	1	2,673,201
	1 – 12	1	1,337,773 – 1,588,121
	13 – 120	5,902 – 5,928	1,337,679 – 1,337,680
USA Road	0 – 60	4,762 – 4,763	392,579 – 392,580
Eagle Peak	0 – 60	3,564 – 3,569	295,569 – 295,570

Figure 9: # of Blocks/Vertices Per Worker (GVD Partitioner)

	WebBase	LiveJournal	BTC	USA	Euro
Coarsen	3547 s	1234 s	4463 s	228 s	184 s
Partition	395 s	836 s	2360 s	8 s	5 s
Uncoarsen	1414 s	205 s	1064 s	171 s	156 s
IdRecode	94 s	24 s	107 s	25 s	24 s
Total	5450 s	2299 s	7994 s	432 s	369 s

Figure 11: Partitioning Performance of Giraph++

	WebUK	WebBase	Friendster	LiveJournal	BTC	USA	Euro
Runtime	4863 s	1373 s	3547 s	205 s	1394 s	127 s	92s

Figure 12: Partitioning Performance of LDG

Comparison with existing partitioning methods. One of the widely used graph partitioning algorithms is METIS [7] (e.g., GRACE [20] uses METIS to partition the input graph). However, METIS ran out of memory on those large graphs in Figure 7 on our platform. To solve the scalability problem of METIS, Giraph++ [18] proposed a graph coarsening method to reduce the size of the input graph so that METIS can run on the smaller graph. Here, a vertex in the coarsened graph corresponds to a set of connected vertices in the original graph. The partitioning algorithm of Giraph++ consists of 4 phases: (1)graph coarsening, (2)graph partitioning (using METIS), (3)graph uncoarsening, which projects the block information back to the original graph, and (4)ID recoding, which relabels the vertex IDs so that the worker of a vertex v can be obtained by hashing v's new ID. Note that ID recoding is not required in Blogel since the worker ID of each vertex v is stored in its triplet ID $trip(v)$. This approach retains the original vertex IDs so that Blogel's graph computing results require no ID re-projection. Blogel's graph partitioning is also more user-friendly, since it requires only one partitioner job, while Giraph++'s graph partitioning consists of a sequence of over 10 Giraph/MapReduce/METIS jobs.

Figure 11 shows the partitioning performance of Giraph++, where we ran as many workers per machine as possible (without running out of memory). We did not obtain result for *WebUK* and *Friendster*, since graph coarsening ran out of memory even when each slave machine runs only one worker. Figure 11 shows that the partitioning time of Giraph++ is much longer than that of our GVD partitioner. For example, while our GVD partitioner partitions *WebBase* in 334.94 seconds (see the breakdown time in Figure 8), Giraph++ uses 5450 seconds. In general, our GVD partitioner is tens of times faster than Giraph++'s METIS partitioning algorithm.

Recently, Stanton and Kliot [17] proposed a group of algorithms to partition large graphs, and the best one is a semi-streaming algorithm called Linear (Weighted) Deterministic Greedy (*LDG*). We also ran LDG and the results are presented in Figure 12. We can see that LDG is many times slower than our GVD partitioner (reported in Figure 8), though LDG is much faster than Giraph++'s new METIS partitioning algorithm.

8.3 Partitioner Scalability

We now study the scalability of our GVD partitioner. We first test the partitioning scalability using two real graphs: *BTC* with skewed degree distribution, and *USA* with a large graph diameter. We partition both graphs using varying number of slave machines (i.e., 6, 9, 12 and 15), and study how the partitioning performance scales with the amount of computing resources. We report the results in Figure 13. For the larger graph *BTC*, we need a certain amount of computing resources to achieve good performance. For example, when the number of slave machines increases from 6 to 9, the partitioning time of *BTC* improves by 25.7%, from 189.2 seconds to 140.52 seconds. However, the performance does not further improve if we increase the number of machines beyond 12. This is because the increased overhead of inter-machine communication outweighs the increased computing power. For the relatively smaller graph *USA*, the performance does not change much with varying number of slave machines, since the computing power is sufficient even with only 6 slaves.

To test the scalability of our GVD partitioner as the graph size increases, we generate random graphs using PreZER algorithm [12]. We set the average degree as 20 and vary $|V|$ to be 25M, 50M, 75M and 100M. Figure 14 shows the scalability results, where all the 16 machines in our cluster are used. The partitioning time increases almost linearly with $|V|$, which verifies that our GVD partitioner scales well with graph size. Moreover, even for a graph with $|V| = 100M$ (i.e., $|E| \approx 2B$), the partitioning is done in

		Machine #			
		6	9	12	15
BTC	Load	60.35 s	38.50 s	32.00 s	28.71 s
	Compute	189.20 s	140.52 s	119.53 s	115.52 s
	Dump	59.13 s	52.96 s	49.65 s	53.66 s
	Total	308.69 s	231.98 s	201.18 s	197.90 s
USA	Load	15.40 s	13.01 s	10.37 s	6.57 s
	Compute	40.05 s	41.32 s	40.40 s	44.74 s
	Dump	6.46 s	4.66 s	4.46 s	3.95 s
	Total	61.91 s	58.99 s	55.23 s	55.26 s

Figure 13: GVD Partitioner Scalability on Real Graphs

	$\|V\|$ (avg deg = 20)			
	25M	50M	75M	100M
Load	11.44 s	29.05 s	42.10 s	74.31 s
Compute	45.32 s	88.76 s	127.32 s	164.13 s
Dump	22.26 s	41.64 s	58.86 s	76.84 s
Total	79.02 s	159.45 s	228.29 s	315.28 s

Figure 14: GVD Partitioner Scalability on Random Graphs

only 164.13 seconds. In contrast, even for the smallest graph with $|V| = 25M$, Giraph++'s new METIS partitioning algorithm cannot finish in 24 hours.

8.4 Performance of Graph Computing

We now report the performance of various graph computing systems for computing CC, SSSP, reachability, and PageRank. We run the vertex-centric algorithms of Blogel (denoted by *V-centric*), as well as the block-centric algorithm of Blogel (denoted by *B-GVD*, *B-2D* or *B-URL* depending on which partitioner is used). Note that *B-2D* applies to road networks only, *B-URL* applies to web graphs only, while *B-GVD* applies to general graphs. We compare with Giraph, GraphLab, and the graph-centric system Giraph++. For GraphLab, we use its synchronous mode since this paper focuses on synchronous computing model. For Giraph++, we do not report the results for *WebUK* and *Friendster* since Giraph++ failed to partition these large graphs.

Figure 15(a) shows the results of CC computation on three representative graphs: *BTC* (skewed degree distribution), *Friendster* (relatively high average degree), and *USA* (large diameter). We obtain the following observations. First, *V-centric* is generally faster than Giraph and GraphLab, which shows that Blogel is more efficient than existing systems even for vertex-centric computing. Second, *B-GVD* (or *B-2D*) is tens of times faster than *V-centric*, which shows the superiority of our block-centric computing. Finally, *B-GVD* (or *B-2D*) is 1–2 orders of magnitude faster than Giraph++; this is because Blogel's block-centric algorithm works in B-mode where blocks communicates with each other directly, while Giraph++'s graph-centric paradigm does not support B-mode and communication still occurs between vertices.

Figure 15(b) shows the results of SSSP computation on two weighted road network graphs. We see that both *B-GVD* and *B-2D* are orders of magnitude faster than *V-Centric*, which can be explained by the huge difference in the number of supersteps taken by the different models. Giraph++ is also significantly faster than *V-Centric*, but it is still much slower than our *B-2D*. This result verifies that our block-centric model can effectively deal with graphs with large diameter. The result also shows that 2D partitioner allows more efficient block-centric parallel computing than GVD partitioner for spatial networks.

Figure 15(c) shows the results of reachability computation on the small-diameter graph, *LiveJoural*, and the large-diameter graphs, *WebUK* and *USA*. We set the source s to be a vertex that can reach most of the vertices in the input graph and set $t = -1$, which means that the actual computation is BFS from s. As Figure 15(c) shows, our block-centric system, *B-GVD* or *B-2D*, is one to two orders of magnitude faster than the vertex-centric systems, *V-Centric*, Giraph and GraphLab, for processing the large-diameter graphs. But the improvement is limited for the small-diameter graph, since *LiveJoural* is not very large and the vertex-centric systems are already very fast on a small-diameter graph of medium size. Compared with Giraph++, *B-GVD* is significantly more efficient for processing *LiveJoural* and *B-2D* is much faster for processing *USA*, while Giraph++ failed to run on *WebUK*.

For PageRank, we use the two web graphs *WebUK* and *WebBase*. We use both the URL-based partitioner and the GVD partitioner for graph partitioning. Figure 16(d) shows the number of blocks/vertices per worker using URL partitioning, where we see that URL partitioning achieves more balanced workload and less number of blocks than GVD partitioning (cf. Figure 9). This shows that background knowledge about the graph data can usually offer a higher quality partitioning than a general method.

We report the time of computing local PageRank and BlockRank by Blogel's B-mode in Figure 16(a). We present the average time of a superstep and the total number of supersteps of computing PageRank using different systems in Figure 16(b). We also run Blogel's block-centric V-mode with the input graph partitioned by LDG partitioning [17], denoted by *B-LDG* in Figure 16(b). We remark that LDG cannot be used in Blogel's VB-mode and B-mode, since a partition obtained by LDG is not guaranteed to be connected.

The results show that though running V-mode, block-centric computing is still significantly faster than vertex-centric computing (i.e., *V-Centric*, Giraph and GraphLab). Note that *B-GVD* and *B-URL* are also running block-centric V-mode in Figure 16(b). Thus, the result also reveals that our GVD partitioner leads to more efficient distributed computing than LDG. *B-URL* is comparable with *B-LDG* on *WebUK*, but is significantly faster than *B-LDG* on *WebBase*. The superior performance of *B-GVD* and *B-URL* is mainly because the GVD and URL partitioners achieve greater reduction in the number of cross-worker edges than LDG, which results in less number of messages exchanged through the network.

We also notice that the number of supersteps of the block-centric algorithm is more than that of the vertex-centric algorithm, which is mainly due to the fact that the PageRank initialization formula of [6], i.e., $pr(v) = lpr(v) \cdot br(block(v))$, is not effective. We may improve the algorithm by specifying another initialization formula, but this is not the focus of this paper.

Another kind of PageRank algorithm is adopted in Giraph++'s paper [18], which is based on the accumulative iterative update approach of [22]. To make a fair comparison with Giraph++, we also developed a Blogel vertex-centric counterpart, and a block-centric counterpart that runs in VB-mode. As Figure 16(c) shows, Blogel's block-centric computing (i.e., *B-GVD* or *B-URL*) is significantly faster than its vertex-centric counterpart (i.e., *V-Centric*) when accumulative iterative update is applied. We can only compare with Giraph++ on *WebBase* since Giraph++ failed to partition *WebUK*. Figure 16(c) shows that while Giraph++ is twice faster than *V-Centric*, it is still much slower than *B-GVD* and *B-URL*.

9. CONCLUSIONS

We presented a block-centric framework, called Blogel, and showed that Blogel is significantly faster than existing distributed graph computing systems [1, 10, 4, 18], for processing large graphs with adverse graph characteristics such as skewed degree distribution, high average degree, and large diameter. We also showed that Blogel's partitioners generate high-quality blocks and are much faster than the state-of-the-art graph partitioning methods [17, 18].

		Load	Compute	Step #	Dump
BTC	V-Centric	24.22 s	28.48 s	30	8.36 s
	B-GVD	*7.58 s*	*0.94 s*	6	*6.16 s*
	Giraph	70.26 s	94.54 s	30	17.14 s
	Giraph++	102.01 s	101.29 s	5	25.64 s
	GraphLab	105.48 s	83.1 s	30	19.03 s
Friendster	V-Centric	82.68 s	120.24 s	22	*1.55 s*
	B-GVD	*16.08 s*	*2.52 s*	6	2.31 s
	Giraph	95.88 s	248.29 s	22	6.37 s
	GraphLab	188.59 s	77.0 s	22	7.57 s
USA Road	V-Centric	5.98 s	510.98 s	6262	0.57 s
	B-GVD	1.47 s	13.95 s	164	*1.00 s*
	B-2D	*1.41 s*	*1.94 s*	26	1.07 s
	Giraph	14.07 s	9518.99 s	6262	2.14 s
	Giraph++	16.81 s	24.00 s	12	2.54 s
	GraphLab	18.27 s	2982.3 s	6262	3.41 s

(a) Performance of Hash-Min

		Load	Compute	Step #	Dump
USA Road	V-Centric	7.21 s	2832.26 s	10789	*1.81 s*
	B-GVD	1.87 s	118.75 s	751	2.46 s
	B-2D	*1.65 s*	*11.29 s*	59	2.58 s
	Giraph	16.68 s	11116.90 s	10789	4.54 s
	Giraph++	18.29 s	80.53 s	39	2.88 s
	GraphLab	19.38 s	9293 s	10789	4.02 s
Euro Road	V-Centric	5.47 s	708.56 s	6210	1.01 s
	B-GVD	1.61 s	68.56 s	440	1.74 s
	B-2D	*1.26 s*	*8.86 s*	55	1.85 s
	Giraph	12.51 s	12712.06 s	6210	*0.05 s*
	Giraph++	18.57 s	87.73 s	30	3.59 s
	GraphLab	16.48 s	3231.3 s	6210	3.17 s

(b) Performance of SSSP

		Load	Compute	Step #	Dump
WebUK	V-Centric	*71.89 s*	144.29 s	664	*0.89 s*
	B-GVD	100.72 s	*41.58 s*	71	2.64 s
	Giraph	530.78 s	1078.06 s	664	13.75 s
	GraphLab	435.95 s	424.2 s	664	15.12 s
Live-Journal	V-Centric	8.93 s	6.87 s	19	*0.37 s*
	B-GVD	*5.54 s*	*4.84 s*	6	0.65 s
	Giraph	37.77 s	17.89 s	19	1.6 s
	Giraph++	17.50 s	29.12 s	4	16.13 s
	GraphLab	16.46 s	8.9 s	19	2.15 s
USA Road	V-Centric	5.18 s	389.37 s	6263	*0.45 s*
	B-GVD	2.31 s	33.31 s	246	0.73 s
	B-2D	*1.51 s*	*4.02 s*	26	1.40 s
	Giraph	16.27 s	5866.19 s	6263	2.64 s
	Giraph++	18.43 s	34.43 s	18	2.51 s
	GraphLab	18.86 s	1558.6 s	6263	3.56 s

(c) Performance of Reachability

Figure 15: Performance of CC, SSSP and Reachability Computation

		Load	Compute *lpr*(*v*) & *br*(*b*)	Dump
WebUK	B-GVD	105.10 s	44.98 s	385.45 s
	B-URL	124.52 s	17.39 s	395.93 s
WebBase	B-GVD	23.93 s	16.68 s	53.79 s
	B-URL	20.27 s	40.32 s	45.36 s

(a) Performance of BlockRank Computation

		Load	Step #	Per-step Time	Dump
WebUK	V-Centric	*111.45 s*	92	31.16 s	*1.99 s*
	B-GVD	113.63 s	92	16.89 s	4.94 s
	B-URL	120.74 s	92	*9.30 s*	4.87 s
WebBase	V-Centric	30.74 s	92	16.53 s	*2.54 s*
	B-GVD	*25.38 s*	92	4.00 s	4.61 s
	B-URL	25.83 s	92	*1.20 s*	4.93 s
	Giraph++	149.49 s	92	7.02 s	26.47 s

(c) Performance of Giraph++'s Version

		Load	Step #	Per-Step Time	Dump
WebUK	V-Centric	71.37 s	89	29.99 s	4.16 s
	B-LDG	104.59 s	89	24.65 s	*2.04 s*
	B-GVD	*47.21 s*	95	*18.63 s*	6.59 s
	B-URL	64.62 s	93	25.96 s	7.08 s
	Giraph	163.99 s	89	53.74 s	22.35 s
	GraphLab	245.62 s	89	48.43 s	16.34 s
WebBase	V-Centric	20.81 s	80	16.23 s	*2.77 s*
	B-LDG	28.30 s	80	9.64 s	3.62 s
	B-GVD	11.51 s	90	4.99 s	6.92 s
	B-URL	*6.39 s*	84	*2.86 s*	5.15 s
	Giraph	61.41 s	80	12.67 s	16.30 s
	GraphLab	79.91 s	80	20.04 s	14.92 s

(b) Performance of PageRank Computation

	Worker	Block #	Vertex #
WebUK	0 – 150	1,693 – 1697	884,987 – 884,988
WebBase	0 – 150	4,930 – 4,931	782,398 – 782,399

(d) # of Blocks/Vertices Per Worker (URL Partitioning)

Figure 16: Performance of PageRank Computation

For future work, we plan to define a class of algorithms similar to PPA [21] for the block-centric computing model.

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