Stratification Criteria and Rewriting Techniques for Checking Chase Termination

S. Greco, F. Spezzano and I. Trubitsyna

DEIS, Università della Calabria - Italy

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1. Motivations
2. Chase Termination Criteria
3. Constraints Rewriting Technique
4. Conclusions

Chase Algorithm

- Sufficient chase termination condition

Example

- $D = \{N(a), S(a)\}$

$$\Sigma = \begin{array}{l} \forall x \; [\; N(x) \rightarrow \exists y \; E(x,y) \;] \\ \forall (x,y) \; [\; S(x) \wedge E(x,y) \rightarrow N(y) \;] \end{array}$$

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- $chase(D,\Sigma) = \{N(a), S(a), E(a,\eta_1), N(\eta_1), E(\eta_1,\eta_2)\}$

- The chase produces a *universal solution*

Chase Algorithm

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- $chase(D, \Sigma) = \{N(a), S(a), E(a, \eta_1), N(\eta_1), E(\eta_1, \eta_2)\}$

- Areas of application: query optimization, data exchange, consistent query answering,. . .

A Non-terminating Chase Sequence

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- Chase termination: undecidable problem (Deutsch et al. [2008])

Weak Acyclicity

(Dependency Graph)

- represent the flow of values
- nodes are positions
- special edges $\xrightarrow{*}$ indicates the generation of new null values

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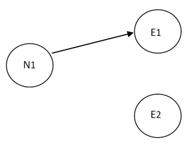

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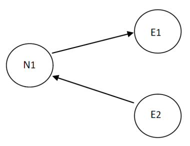

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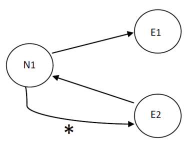
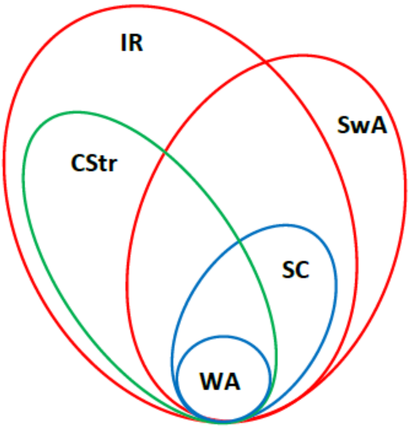


$\mathcal{WA}$, $\mathcal{SC}$, $\mathcal{CS}tr$, $\mathcal{IR}$ and $\mathcal{S}w\mathcal{A}$ Relationships



Contributions

- Introduction of new chase termination criteria
  - *WA*-stratification (*SC*-stratification and *SwA*-stratification)
  - Local Stratification
- New rewriting technique improved by analyzing affected positions

Chase Termination Criteria

C-Stratification [DeutschNR08,MeierSL09]

- Builds a *c-chase graph* $G_c(\Sigma) = (\Sigma, E)$ representing how constraints fire each other.
- $(r_1, r_2) \in E$ if $r_1 \prec_c r_2$ (firing r_1 can cause r_2 to fire)
- $r_1 \prec_c r_2$ if:
  1) $K_1^{*, r_1} \xrightarrow{h_1} K_2$,
  2) $K_2 \not\models h_2(r_2)$,
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C-Stratification [DeutschNR08,MeierSL09]

(C-Stratification)

A set of constraint Σ is c-stratified if every cycle in the c-chase graph is weakly acyclic.

Example

$$\Sigma = \{r : T(x,y) \wedge T(y,x) \rightarrow \exists z\ T(y,z) \wedge T(z,x)\}$$

we have that $r \prec_c r$ and $\{r\}$ is not weakly acyclic

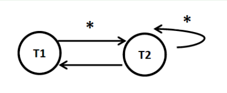

Figure: Dependency graph $Dep(\Sigma)$

C-Stratification versus Stratification

- C-Stratification

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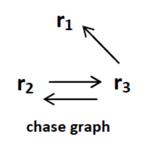
- Stratification guarantees the termination of at least one chase sequence.

Example: Stratification

Example

$$\Sigma = \begin{array}{l} r_1 : \; R(x) \rightarrow \exists y \; T(x,y) \\ r_2 : \; R(x) \rightarrow T(x,x) \\ r_3 : \; T(x,y) \wedge T(x,x) \rightarrow R(y) \end{array}$$



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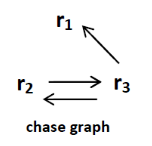

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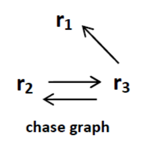

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  1. if $K_1 = \{R(a)\}$ then $K_2 = \{R(a), T(a, \eta_1)\} \models r_3$ (because $T(a,a) \notin K_2$)

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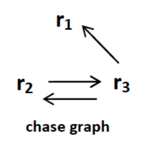

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  2. if $K_1 = \{R(a), T(a,a)\}$ then $K_1 \models \Sigma$ (K_1 universal solution)

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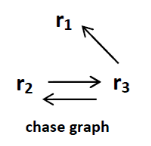

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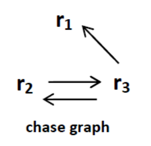

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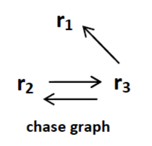

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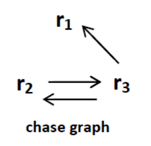

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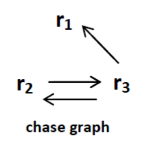

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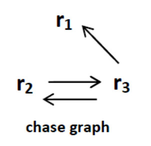

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$D_3 \xrightarrow{r_1, h_4} D_4 = \{R(a), T(a,\eta_1), T(a,a), R(\eta_1), T(\eta_1,\eta_2)\}$

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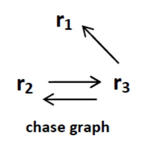

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  $\ldots$

- chase sequence $r_2 \rightarrow r_3 \rightarrow r_1$ terminates

WA-stratification

- Builds a *firing graph* $\Gamma(\Sigma) = (\Sigma, E)$ representing how constraints fire each other.
- $(r_1, r_2) \in E$ if $r_1 < r_2$ (firing r_1 can cause r_2 to fire)
- $r_1 < r_2$ if:

  1) $K_1 \stackrel{r_1, h_1}{\rightarrow} K_2$,

  2) $K_2 \cup S \not\models h_2(r_2)$,

  3) $K_1 \cup S \models h_2(r_2)$ and

  4) $Null(S) \cap (Null(K_2) - Null(K_1)) = \emptyset$.

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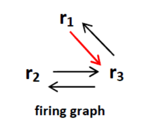
- $K_1 = \{R(a)\}$ and $K_2 = \{R(a), T(a, \eta_1)\}$
- $S = \{T(a,a)\}$
- $r_3: \; T(a, \eta_1) \wedge T(a,a) \rightarrow R(\eta_1)$
- r_3 is fired by r_1, then we have $r_1 < r_3$



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(WA-Stratification)

We say that Σ is *WA-stratified* (*WAStr*) iff the constraints in every nontrivial strongly connected component of the *firing graph* $\Gamma(\Sigma) = (\Sigma, \{(r_1, r_2) | r_1 < r_2\})$ are weakly acyclic.

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Example

$$\Sigma = \{r : T(x, y) \wedge T(y, x) \rightarrow \exists z\ T(y, z) \wedge T(z, x)\}$$

is *WA*-stratified, but not c-stratified.

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We say that Σ is *WA-stratified* (*WAStr*) iff the constraints in every nontrivial strongly connected component of the *firing graph* $\Gamma(\Sigma) = (\Sigma, \{(r_1, r_2) | r_1 < r_2\})$ are weakly acyclic.

Example

$$\Sigma = \{r : T(x, y) \wedge T(y, x) \rightarrow \exists z\ T(y, z) \wedge T(z, x)\}$$

is *WA*-stratified, but not c-stratified.

Proposition

$\mathcal{CS}tr \subsetneq \mathcal{WAS}tr$ and $\mathcal{SC} \nparallel \mathcal{WAS}tr$.

SC-stratification and *SwA*-stratification

Definition

Given a set of TDGs Σ, we say that Σ is

- *SC-stratified* (*SCStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are safe.
- *SwA-stratified* (*SwAStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are super-weak acyclic.

SC-stratification and *SwA*-stratification

Definition

Given a set of TDGs Σ, we say that Σ is

- *SC-stratified* (*SCStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are safe.
- *SwA-stratified* (*SwAStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are super-weak acyclic.

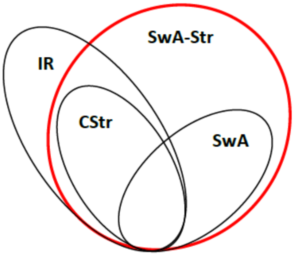
Proposition

Let Σ be a set of WA-stratified (resp. SC-stratified, SwA-stratified) TGDs and let D be a database. Then, the length of every chase sequence of Σ over D is polynomial in the size of D.

SwA-stratification

Theorem

1. $\mathcal{WAS}tr \subsetneq \mathcal{SCS}tr \subsetneq \mathcal{S}w\mathcal{AS}tr$,
2. for $C \in \{WA, SC, SwA\}$, $\mathcal{C} \subsetneq \mathcal{C}\text{-}\mathcal{S}tr$ and
3. $\mathcal{IR} \nparallel \mathcal{S}w\mathcal{AS}tr$.



Local Stratification

New chase termination condition: **Local Stratification (LS)**

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- Combine $<$-relation of $\mathcal{WA}$-$\mathcal{S}tr$, with *SwA* criterion,

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- Combine $<$-relation of $\mathcal{WA}$-$\mathcal{S}tr$, with *SwA* criterion,
- ensures the termination of all chase sequences in polynomial data complexity,
- $\mathcal{S}w\mathcal{A}$-$\mathcal{S}tr \subsetneq \mathcal{LS}$ and $\mathcal{IR} \subsetneq \mathcal{LS}$.

Example: Super-weak Acyclicity [Marnette09]

Analyzes nulls propagation through constraints

Example

$$\Sigma = \begin{array}{ll} r_1: & N(x) \rightarrow \exists y\, \exists z\, E(x,y) \wedge S(z,y) \\ r_2: & E(x_1,y_1) \wedge S(x_1,y_1) \rightarrow N(y_1) \\ r_3: & E(x_2,y_2) \rightarrow E(y_2,x_2) \end{array}$$

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Then, we say that $r_1 \rightsquigarrow r_1$
because the null introduced by r_1 (p_3 and p_5) is copied again in the head of r_1 (p_2)

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The relation $\rightsquigarrow$ is cyclic and Σ is not in SwA

Local Stratification

IDEA: testing the firing relation $<$ from *WA*-stratification in the *MOVE* construction

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$$\Sigma = \begin{array}{ll} r_1: & N(\overset{p_1}{x}) \rightarrow \exists y\, \exists z\, E(\overset{p_2}{x}, \overset{p_3}{y}) \wedge S(\overset{p_4}{z}, \overset{p_5}{y}) \\ r_2: & E(\overset{p_6}{x_1}, \overset{p_7}{y_1}) \wedge S(\overset{p_8}{x_1}, \overset{p_9}{y_1}) \rightarrow N(\overset{p_{10}}{y_1}) \\ r_3: & E(\overset{p_{11}}{x_2}, \overset{p_{12}}{y_2}) \rightarrow E(\overset{p_{13}}{y_2}, \overset{p_{14}}{x_2}) \end{array}$$

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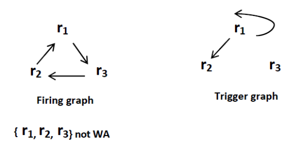

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$r_1 \not< r_2$

$MOVE(r_1, y) = \{p_3, p_5\}$

$p_{10} \notin MOVE(r_1, y)$ as $r_1 \not< r_2$

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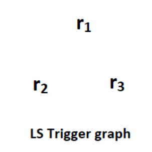
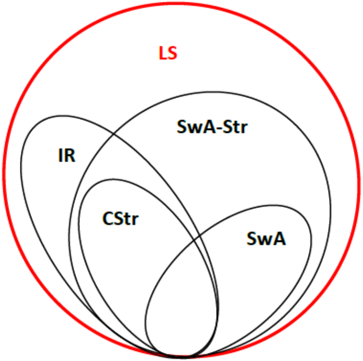
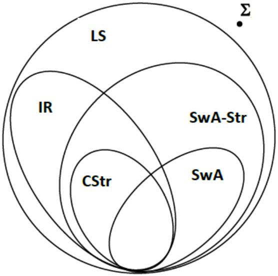


Local Stratification



Limitations



Example

$$\Sigma = \begin{array}{l} \forall x \; [\; N(x) \rightarrow \exists y \; E(x,y) \;] \\ \forall (x,y) \; [\; S(x) \wedge E(x,y) \rightarrow N(y) \;] \end{array}$$

Constraints Rewriting Technique

Constraints Rewriting Technique [GrecoS10]

- Idea: rewrite Σ into an 'equivalent' set Σ^α and verify the structural properties for chase termination on Σ^α
  - introduction of adornments associated with predicates

Adn(∑)

In(∑)		Base(∑)		Derived(∑)		Out (∑)
$p \rightarrow p^{b\ldots b}$	$\cup$	$p^{b\ldots b} \rightarrow p^\alpha$	$\cup$	$p^\alpha \rightarrow p^\beta$	$\cup$	$p^\gamma \rightarrow \hat{p}$

$$\equiv$$

Map(∑)

$$\Sigma \;\cup\; p \rightarrow \hat{p}$$

Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} N(x) \rightarrow \exists y\ E(x,y) \\ S(x) \wedge E(x,y) \rightarrow N(y) \end{array}$$

Rewriting Algorithm

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- $In(\Sigma) =$
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  -
  -

Rewriting Algorithm

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  - $N^f(x) \rightarrow \exists y\ E^{ff}(x,y)$

Rewriting Algorithm

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  $N^b(x) \rightarrow \hat{N}(x)$
  $N^f(x) \rightarrow \hat{N}(x)$
  $S^b(x) \rightarrow \hat{S}(x)$
  $E^{bb}(x,y) \rightarrow \hat{E}(x,y)$
  $E^{bf}(x,y) \rightarrow \hat{E}(x,y)$
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Rewriting Algorithm

Example

$$D = \{N(a), S(a)\}$$

$chase(D, Map(\Sigma)) = \{$

$N(a)\ S(a)\ E(a, \eta_1)$
$N(\eta_1)\quad E(\eta_1, \eta_2)$

$\hat{N}(a)\ \hat{S}(a)\ \hat{E}(a, \eta_1)$
$\hat{N}(\eta_1)\quad \hat{E}(\eta_1, \eta_2)$
$\}$

$chase(D, Adn(\Sigma)) = \{$

$N(a)\ S(a)$

$N^b(a)\ S^b(a)\ E^{bf}(a, \eta_1)$
$N^f(\eta_1)\quad E^{ff}(\eta_1, \eta_2)$

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$\}$

Chase Termination Checking

- Test chase termination criteria over rewritten constraints $Adn(\Sigma)$

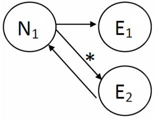


Figure: $Dep(\Sigma)$

- $Adn(\Sigma) = \{$
  $\dots$
  $N^b(x) \to \exists y\ E^{bf}(x,y)$
  $S^b(x) \wedge E^{bb}(x,y) \to N^b(y)$
  $S^b(x) \wedge E^{bf}(x,y) \to N^f(y)$
  $N^f(x) \to \exists y\ E^{ff}(x,y)$
  $\dots\}$

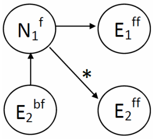


Figure: $Dep(Adn(\Sigma))$

Chase Termination Checking

- *Adn*-$\mathcal{T}$: class of sets of TGDs Σ s.t. *Adn*(Σ) is in $\mathcal{T}$

Theorem

$\mathcal{T} \subsetneq Adn\text{–}\mathcal{T}$ for $\mathcal{T} \in \{\mathcal{WA}, \mathcal{SC}, \mathcal{CS}tr, \mathcal{IR}, \mathcal{S}w\mathcal{A}, \mathcal{WAS}tr, \ldots, \mathcal{LS}\}$.

- Rewriting technique is orthogonal to termination conditions
- improves current chase termination criteria

Constraints Rewriting Technique

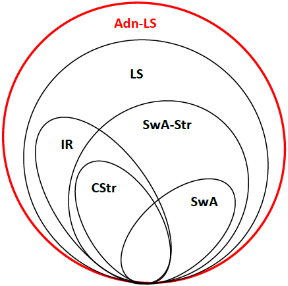


Figure: Criteria Relationships

Improving Rewriting Algorithm

Adn^{++}

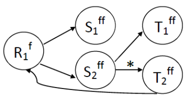
A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{ll} r_1: & R(x) \rightarrow S(x,x) \\ r_2: & S(x_1,x_2) \rightarrow \exists z\, T(x_2,z) \wedge Q(x_2) \\ r_3: & T(x_1,x_2) \wedge T(x_1,x_3) \wedge T(x_3,x_1) \rightarrow R(x_2) \end{array}$$

- $Adn(\Sigma) \notin \mathcal{LS}$

$$Adn(\Sigma) = \begin{array}{l} \ldots \\ R^f(x) \rightarrow S^{ff}(x,x) \\ S^{ff}(x_1,x_2) \rightarrow \exists z\, T^{ff}(x_2,z) \wedge Q^f(x_2) \\ T^{ff}(x_1,x_2) \wedge T^{ff}(x_1,x_3) \wedge T^{ff}(x_3,x_1) \rightarrow R^f(x_2) \end{array}$$



- $chase(D,\Sigma)$ terminates for all database D

A New Rewriting Algorithm

Adn^{++} algorithm improves Adn

A New Rewriting Algorithm

Adn^{++} algorithm improves *Adn*

- testing of the $<$-relation during the generation of adorned constraints,

A New Rewriting Algorithm

Adn^{++} algorithm improves *Adn*

- testing of the $<$-relation during the generation of adorned constraints,
- the adornment f_i is assigned in a more refined way (skolem function),

A New Rewriting Algorithm

Adn^{++} algorithm improves *Adn*

- testing of the $<$-relation during the generation of adorned constraints,
- the adornment f_i is assigned in a more refined way (skolem function),
- cyclicity detection (Adn^{++} returns a boolean value, *cyc*, saying if there is cyclicity among the adorned constraints).

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y\, E(x, y) \\ r_2 : \ E(x, y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{ll} r_1 : & N(x) \rightarrow \exists y\, E(x,y) \\ r_2 : & E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{ll} r_1: & N(x) \rightarrow \exists y\, E(x,y) \\ r_2: & E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i: \; N^b(x) \rightarrow \exists y\, E^{bf_1}(x,y)$

$r_2^i: \; E^{bb}(x,y) \rightarrow N^b(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y \ E(x,y) \\ r_2 : \ E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \ N^b(x) \rightarrow \exists y \ E^{bf_1}(x,y)$

$r_2^i : \ E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

A New Rewriting Algorithm

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$r_2^i : E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

A New Rewriting Algorithm

Example

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$Derived(\Sigma) =$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2 : \; E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \; N^b(x) \rightarrow \exists y \; E^{bf_1}(x,y)$

$r_2^i : E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2 : \; E(x,y) \rightarrow N(y) \end{array}$$

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$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y\, E(x,y) \\ r_2 : \ E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : N^b(x) \rightarrow \exists y\ E^{bf_1}(x,y)$

$r_2^i : \ E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \ E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \to \exists y \; E(x,y) \\ r_2 : \; E(x,y) \to N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : N^b(x) \to \exists y \; E^{bf_1}(x,y)$

$r_2^i : \; E^{bb}(x,y) \to N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \to N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \to \exists y \; E^{f_1 f_2}(x,y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2 : \; E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \; N^b(x) \rightarrow \exists y \; E^{bf_1}(x,y)$

$r_2^i : \; E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \rightarrow \exists y \; E^{f_1 f_2}(x,y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2 : \; E(x,y) \rightarrow N(y) \end{array}$$

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$r_2^i : \; E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \rightarrow \exists y \; E^{f_1 f_2}(x,y)$

A New Rewriting Algorithm

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$Base(\Sigma) =$

$r_1^i : \; N^b(x) \rightarrow \exists y \; E^{bf_1}(x,y)$

$r_2^i : \; E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \rightarrow \exists y \; E^{f_1 f_2}(x,y)$

$r_2^{iii} : \; E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2 : \; E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \; N^b(x) \rightarrow \exists y \; E^{bf_1}(x,y)$

$r_2^i : \; E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \rightarrow \exists y \; E^{f_1 f_2}(x,y)$

$r_2^{iii} : \; E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1: \; N(x) \rightarrow \exists y \; E(x,y) \\ r_2: \; E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

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$Base(\Sigma) =$

$r_1^i : \; N^b(x) \rightarrow \exists y \; E^{bf_1}(x,y)$

$r_2^i : \; E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \; E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \; N^{f_1}(x) \rightarrow \exists y \; E^{f_1 f_2}(x,y)$

$r_2^{iii} : \; E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

$r_1^{iii} : \; N^{f_2}(x) \rightarrow \exists y \; E^{f_2 f_3}(x,y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y\ E(x,y) \\ r_2 : \ E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \ N^b(x) \rightarrow \exists y\ E^{bf_1}(x,y)$

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$Derived(\Sigma) =$

$r_2^{ii} : \ E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \ N^{f_1}(x) \rightarrow \exists y\ E^{f_1 f_2}(x,y)$

$r_2^{iii} : \ E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

$r_1^{iii} : \ N^{f_2}(x) \rightarrow \exists y\ E^{f_2 f_3}(x,y)$

$r_2^{iv} : \ E^{f_2 f_3}(x,y) \rightarrow N^{f_3}(y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y\ E(x,y) \\ r_2 : \ E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$Base(\Sigma) =$

$r_1^i : \ N^b(x) \rightarrow \exists y\ E^{bf_1}(x,y)$

$r_2^i : \ E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \ E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \ N^{f_1}(x) \rightarrow \exists y\ E^{f_1 f_2}(x,y)$

$r_2^{iii} : \ E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

$r_1^{iii} : \ N^{f_2}(x) \rightarrow \exists y\ E^{f_2 f_3}(x,y)$

$r_2^{iv} : \ E^{f_2 f_3}(x,y) \rightarrow N^{f_3}(y)$

$N^{f_3}(x) \rightarrow \exists y\ E^{f_3 f_4}(x,y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : N(x) \rightarrow \exists y\, E(x,y) \\ r_2 : E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = false$

$\theta = \{f_3/f_1, f_4/f_2\}$

$Base(\Sigma) =$

$r_1^i : N^b(x) \rightarrow \exists y\, E^{bf_1}(x,y)$

$r_2^i : E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : N^{f_1}(x) \rightarrow \exists y\, E^{f_1 f_2}(x,y)$

$r_2^{iii} : E^{f_1 f_2}(x,y) \rightarrow N^{f_2}(y)$

$r_1^{iii} : N^{f_2}(x) \rightarrow \exists y\, E^{f_2 f_3}(x,y)$

$r_2^{iv} : E^{f_2 f_3}(x,y) \rightarrow N^{f_3}(y)$

$N^{f_3}(x) \rightarrow \exists y\, E^{f_3 f_4}(x,y)$

A New Rewriting Algorithm

Example

$$\Sigma = \begin{array}{l} r_1 : \ N(x) \rightarrow \exists y \ E(x,y) \\ r_2 : \ E(x,y) \rightarrow N(y) \end{array}$$

$r_1 < r_2$

$r_2 < r_1$

$cyc = true$

$Base(\Sigma) =$

$r_1^i : \ N^b(x) \rightarrow \exists y \ E^{bf_1}(x,y)$

$r_2^i : \ E^{bb}(x,y) \rightarrow N^b(y)$

$Derived(\Sigma) =$

$r_2^{ii} : \ E^{bf_1}(x,y) \rightarrow N^{f_1}(y)$

$r_1^{ii} : \ N^{f_1}(x) \rightarrow \exists y \ E^{f_1 f_2}(x,y)$

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$r_1^{iii} : \ N^{f_2}(x) \rightarrow \exists y \ E^{f_2 f_3}(x,y)$

$r_2^{iv} : \ E^{f_2 f_3}(x,y) \rightarrow N^{f_3}(y)$

$r_1^{iv} : \ N^{f_3}(x) \rightarrow \exists y \ E^{f_1 f_2}(x,y)$

A New Rewriting Algorithm

Lemma

For every set of TGDs Σ the function Adn^{++} always terminates.

Theorem

For every set of TGDs Σ over a database schema $\mathbf{R}$

$$\langle Map(\mathbf{R}), Map(\Sigma)\rangle \equiv_{\mathbf{R}/\hat{\mathbf{R}}} \langle Adn^{++}(\mathbf{R}, \Sigma), Adn^{++}(\Sigma)\rangle$$

Theorem

$Adn\text{-}\mathcal{T} \subsetneq Adn^{++}\text{-}\mathcal{T}$, for
$\mathcal{T} \in \{\mathcal{WA}, \mathcal{SC}, \mathcal{S}tr, \mathcal{CS}tr, \mathcal{S}w\mathcal{A}, \mathcal{WAS}tr, \ldots, \mathcal{LS}\}$.

Acyclic set of TGDs

Definition

A set of TGDs Σ is said to be Acyclic (AC) if cyc is false.

Acyclic set of TGDs

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A set of TGDs Σ is said to be Acyclic (AC) if cyc is false.

Example

$$\Sigma = \begin{array}{ll} r_1: & R(x,z) \rightarrow \exists y\ T(x,y) \\ r_2: & T(x,y) \rightarrow R(x,y) \end{array}$$

$$Adn^{++}(\Sigma) = \begin{array}{lll} r_1': & R^{bb}(x,z) & \rightarrow \exists y\ T^{bf_1}(x,y) \\ r_2': & T^{bb}(x,y) & \rightarrow R^{bb}(x,y) \\ r_2'': & T^{bf_1}(x,y) & \rightarrow R^{bf_1}(x,y) \\ r_1'': & R^{bf_1}(x,z) & \rightarrow \exists y\ T^{bf_1}(x,y) \end{array}$$

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A set of TGDs Σ is said to be Acyclic (AC) if cyc is false.

Example

$$\Sigma =$$

$$\begin{array}{ll} r_1: & R(x,z) \rightarrow \exists y\ T(x,y) \\ r_2: & T(x,y) \rightarrow R(x,y) \end{array}$$

$$Adn^{++}(\Sigma) =$$

$$\begin{array}{llll} r_1': & R^{bb}(x,z) & \rightarrow \exists y\ T^{bf_1}(x,y) \\ r_2': & T^{bb}(x,y) & \rightarrow R^{bb}(x,y) \\ r_2'': & T^{bf_1}(x,y) & \rightarrow R^{bf_1}(x,y) \\ r_1'': & R^{bf_1}(x,z) & \rightarrow \exists y\ T^{bf_1}(x,y) \end{array}$$

Criteria Relationships

Theorem

$\mathcal{AC} = Adn{+}{+}\mathcal{WA} = Adn{+}{+}\mathcal{LS}$.

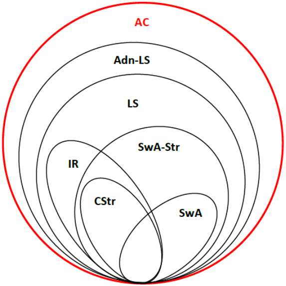

Figure: Criteria Relationships

Conclusions

Conclusions

- We proposed new sufficient chase termination criteria, such as *WA*-Stratification and Local Stratification
- We proposed new rewriting techniques which are orthogonal to termination conditions and improves current chase termination criteria.
- **Current work**: extending Adn^{++} algorithm with EGDs

Thank you!

Any questions?